# EXPERIMENTAL STUDY OF INTERDEPENDENT SYSTEMS: MARKETS AND NETWORKS

**California Institute of Technology**

**AIR FORCE RESEARCH LABORATORY**
**INFORMATION DIRECTORATE**
**ROME RESEARCH SITE**
**ROME, NEW YORK**

This report has been reviewed by the Air Force Research Laboratory, Information Directorate, Public Affairs Office (IFOIPA) and is releasable to the National Technical Information Service (NTIS). At NTIS it will be releasable to the general public, including foreign nations.

AFRL-IF-RS-TR-2002-252 has been reviewed and is approved for publication.

APPROVED:

KEVIN A. KWIAT
Project Engineer

FOR THE DIRECTOR:

WARREN H. DEBANY JR., Technical Advisor
Information Grid Division
Information Directorate

# REPORT DOCUMENTATION PAGE

*Form Approved*
*OMB No. 074-0188*

| 1. AGENCY USE ONLY (Leave blank) | 2. REPORT DATE | 3. REPORT TYPE AND DATES COVERED |
|---|---|---|
| | Sep 02 | Final  Jul 00 – Jan 02 |

**4. TITLE AND SUBTITLE**

EXPERIMENTAL STUDY OF INTERDEPENDENT SYSTEMS:  MARKETS AND NETWORKS

**6. AUTHOR(S)**

Charles R. Plott

**5. FUNDING NUMBERS**
C   - F30602-00-2-0623
PE  - 62301E
PR  - K250
TA  - 03
WU  - 02

**7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)**

California Institute of Technology
Division of Humanities and Social Sciences
CALTECH 228-77
Pasedena, CA 91125

**8. PERFORMING ORGANIZATION REPORT NUMBER**

**9.  SPONSORING / MONITORING AGENCY NAME(S) AND ADDRESS(ES)**

Defense Advanced Research Projects Agency          AFRL/IFGA
3701 North Fairfax Drive                                          525 Brooks Road
Arlington, VA  22203-1714                                      Rome, NY 13441-4505

**10. SPONSORING / MONITORING AGENCY REPORT NUMBER**

AFRL-IF-RS-TR-2002-252

**11. SUPPLEMENTARY NOTES**

**12a. DISTRIBUTION / AVAILABILITY STATEMENT**
Approved for public release; distribution unlimited.

**12b. DISTRIBUTION CODE**

**13. ABSTRACT** *(Maximum 200 Words)*

This report covers some of the first applications of laboratory experimental methods in economics to the problems of understanding the principles that govern adjustments in complex networks of markets exchange processes.  Key experiments were conducted in several areas to determine the appropriateness of classical economics models.  Because the nature of networks suggests the possibility of instability and because there was little previous understanding of how to study instability experimentally, little theory exists and virtually to previous experiments exist.  The first step was to determine if classical theory had any value at all, so a broad investigation was initiated, as opposed to an in-depth study of a narrow issue.  The results are that existing market theory receives considerable support.  System tendencies are to move toward the predictions of equilibrium models so long as the dynamic theory supports such movements.  In this regard a clear need for theories of disequilibrium is evident.  Part of the research focused on what is becoming known as Information Aggregation Mechanisms.   Special networks can be designed to collect "soft" information held in the form of "opinions and intuitions" based on limited but independently acquired observations by decentralized personnel.

**14. SUBJECT TERMS**
Networks, Market-Mechanisms, Resource Allocation

**15. NUMBER OF PAGES**
42

**16. PRICE CODE**

| 17. SECURITY CLASSIFICATION OF REPORT | 18. SECURITY CLASSIFICATION OF THIS PAGE | 19. SECURITY CLASSIFICATION OF ABSTRACT | 20. LIMITATION OF ABSTRACT |
|---|---|---|---|
| UNCLASSIFIED | UNCLASSIFIED | UNCLASSIFIED | UL |

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# ABSTRACT

The paper reports some of the first applications of laboratory experimental methods in economics to the problems of understanding the principles that govern adjustments in complex networks of markets exchange processes. Key experiments were conducted in several areas to determine the appropriateness of classical economics models. Because the nature of networks suggests the possibility of instability and because there was little previous understanding of how to study instability experimentally, little theory exists and virtually no previous experiments exist. The first step was to determine if classical theory had any value at all, so a broad investigation was initiated, as opposed to an in-depth study of a narrow issue. The results are that existing market theory receives considerable support (as opposed to a full game theoretic analysis, which seems to be less applicable in the complex cases). System tendencies are to move toward the predictions of equilibrium models so long as the dynamic theory supports such movements. In this regard a clear need for theories of disequilibrium is evident. Part of the research focused on what is becoming known as Information Aggregation Mechanisms. Special networks can be designed to collect "soft" information held in the form of "opinions and "intuitions" based on limited but independently acquired observations by decentralized personnel. Incentives and the theory of subjective probabilities are used to develop probabilities and predictions. A specially crafted parimutuel system of betting is shown to have the capacity of an information aggregation mechanism.

# Executive Summary

The research used the rapidly developing laboratory experimental method of economics to gain an understanding of the behavior of complex networks of markets and the information that is provided through market behavior.  The ability to judge the speed with which networked markets can deliver and the factors that govern that speed can be of substantial importance.  In addition, the behavior of markets and networked systems are known to convey information about events that is otherwise known only locally. The research focused on the information aspects of networks as well.

Initially, the research involved the development of technologies and methodologies needed to study large, interdependent systems of markets.  The research first involved testing the ability of equilibrium models to predict economic activity in networked systems.  The steps were guided by both theories of equilibrium and theories of disequilibrium behavior. Specifically, examples were chosen for experimentation that were complex but not so complex that theory would not produce clear predictions.  The results demonstrated that classical economic theory produces a coherent explanation of the patterns of adjustments observed.  Two types of interdependence were studied. The first was related to the ability of markets to efficiently coordinate the use of inputs. The second was a chain of markets in which decentralized coordination was a central feature. In both cases the competitive equilibrium model strongly suggests the directions that economic activity will move.

The second step of the research was guided by the results in the first step.  While tendencies toward equilibrium were observed, the markets were not resting at an equilibrium in a game theory sense. The systems seem to be driven by principles of dynamics, so generalizations of classical stability of multiple markets were explored. Earlier experiments have shown that multiple markets systems exhibit some of the starkly surprising properties predicted by classical principles of market adjustments. Specifically, under certain parametric conditions the theory predicts that markets will exhibit a type of orbit behavior and never converge.  Aspects of initial conditions dictate the direction of the orbit.

The issue is especially important for networks. Networked markets have some of the properties that the classical theory of dynamics predicts will lead to non-convergence of prices.  The question posed for experimentation was whether or not this type of dynamics was mitigated by the existence of more complete or orderly forms of market architectures.  Specifically, the issue was whether or not the existence of an order book, which plays a time coordination role in markets, would alter the dynamics.  The expectation is based on the fact that the classical dynamic models are based on strong assumptions of decision maker myopia and the existence of an order book guides decisions into a more forward looking perspective.  Experiments demonstrate that the dynamics predicted by the classical models persists in the more modern market architectures.

Since the first two steps in the research produced support for key aspects of classical theory the next step was to study the ability of the principles to predict when the economic activities were conducted on a larger and more complex scale.  For this exercise an international finance experiment was chosen.  The large scale experiments included the essential economic relationships that were the subject of the experiments conducted at the previous steps of research.  The question posed was whether or not feedback, lags and other forms of market sluggishness would prevent the orderly development of economic activity. For the most part the conclusions developed from the first two phases survived the tests of scale.  The results suggest features that might surface as the scale of operations increases.

The final part of the study is focused on information aggregation.  On the surface it would seem that the problem of designing mechanisms to collect opinions and hunches and integrate them into a prediction would fall outside the realm of the study. However, that is not the case since information processing and features of markets that carry information are key features of market adjustments. Thus the final phase of the study allowed the simultaneous study of information processing by individuals and systems while producing a technology for transforming "soft" information such as opinions into hard information and a prediction.

Special systems of networks have the needed capacities for dealing with decentralized information sources. A new type of parimutuel betting system was designed and tested. This system has the capability of collecting opinions expressed in terms of betting behavior (for real money) and transforming them into a hard prediction with mean predictions and standard errors. Tests of the system under laboratory conditions demonstrate that the system is relatively accurate and when the predictions of the system are unreliable it provides some signals that is the case.

# 1. Task Objectives

The tasks involve the use of laboratory experimental methods in the context of complex and interdependent systems to explore the basic principles of economics and game theory used in popular models. This brings to the scientific discussion a level of precision in measuring the accuracy of models in the context of a challenge that has not existed before.

Theory and limited field experience suggest that decentralized market processes have the capacity to coordinate highly interdependent activities and while doing so transmit information across the system that is otherwise held in a decentralized form. This study explores both the capacity for markets to perform such functions as well as the principles in operation if it takes place. This understanding is sought in order to facilitate greater use of decentralize processes as well as support the designs of mechanism to support them.

Widely applied theory is based on the principle that market processes maintain themselves at the solution to a specific set of equations. The research calls for an examination of this basic, scientific proposition in the context of different types of interdependencies. Five questions are posed.

1. Is it technically possible to create and study complex systems of markets in the context of a laboratory environment?
2. Can multiple market systems accommodate the interdependencies and scale related to decentralized, networked systems? Do complex and interdependent markets exhibit any coherence at all?
3. Is the behavior of interdependent systems approximated by the competitive model?
4. What principles of dynamics are in operation?
5. Does any success in modeling these complex processes continue when the scale is increased?

A closely related issue is the capacity of market-like processes to collect and aggregate information that is otherwise dispersed in a decentralized manner as the "soft information" in the form of opinions and guesses. Established results demonstrate that by using specially designed processes called Information Aggregation Mechanisms (IAM), such information can be collected, aggregated and transformed into "hard" information in the form of probabilities. The question posed for research is:
6. Can a previously studied IAM, one based on principles of parimutuel betting, be substantially improved?

# 2. Technical Approach

## 2.1 Coordination of Networked Markets

The research will first focus on networked markets with the specific characteristic of vertical interdependence that might be a challenging environment for the study of competitive processes and interdependent, competitive markets. These environments will be used to explore the answers to questions two and three that were posed for study.

Since the competitive model is an equilibrium model, with all magnitudes determined simultaneously the natural timing of events that necessarily take place in markets is a natural place to explore the possibility that the model might fail. Specifically, in vertically related markets in which inputs are transformed to outputs, which then become inputs to a new market there is reason to suspect that a simultaneous equation, equilibrium model would fail. In such links of markets output price depends on demand and cost. Cost depends on input prices. Demand for inputs and thus input prices depend on output price. A fundamental circularity exists that might facilitate a lack of coordination.

Two different environments were studied. (1) [Multiple inputs and single output] The first and most elementary involved resource markets and output markets connected by profit seeking producers. It is a standard multiple input, single output world that can be found in any textbook on micro economic theory. The complexity stems from the fact that two inputs markets must be coordinated together with a single output market. While the conception is easy the problem is not, especially given the complex non-linearities that were implemented. (2) [A long chain] The second involves a more complex chain of eight markets. The second could be interpreted as each market being the market for an input to be used in a higher level process in producing an output. An alternative interpretation is as a spatially connected series of markets in which goods must travel through a chain of eight locations in order to get from the original sellers located at the first location to the final buyers located at the last. Transportation between locations is accomplished by a series of middlemen transporters, each of which can operate only between two locations and makes money by buying in one of them, transporting and selling in the other.

## 2.2 Principles of Dynamics and Adjustment

The fourth question posed for study is a focus on the process at work to form prices and allocations as activity takes place in a market. Two basic principles of dynamics are found in models that attempt to ascertain the directions in which markets are likely to move. (1)[ Complements] One has been studied extensively and is known to predict some surprising features of markets, e.g. non convergence or even instability, when activities are complementary (mutually reinforcing). The question posed here is whether or not previous findings are robust when the market institutions are changed to those that are likely to be operating in the field. (2) [Multiple substitutes and entry] The second deals with cases in which entry and exit from a market can take place. From the point of view of models these are cases in which the system is on a "boundary".

## 2.3 The Robustness of Results to Scale

The fifth question posed for research is whether or not the results continue to operate as the scale of the system of markets is increased. To study this, a relatively large (for laboratory experimental work) system of markets was created. The system was based on international trade and finance because of the complex interdependencies that the theory suggests are there. The experiments involved on the order of 60 people participating over a two day period. From an model point of view, if all equations are counted they would number in the hundreds. From an interdependence point of view there are independent networks of markets, each country has its own network of markets, embedded in a network with exchange rates being the connectors. So, there is a network of networks.

## 2.4 Information Aggregation Mechanisms

The sixth question posed for study provides a step in the rapidly developing area of information aggregation. It is the design and implementation of special mechanisms for collecting opinions. The methodology involves the creation of incentives for the expression of opinions and a technology so that the expressions can be transformed into subjective probabilities and aggregated. A particular type of betting system that has market-like features has showed promise. The task involves testing some proposed improvements in the mechanism.

# 3. General Methodology

Laboratory methods from economics were used throughout the research. These methods create operational processes, such as markets, in which various institutional details are present. Subjects operate in such processes, making financially motivated decisions in which the incentives are performance based. Since the incentives and potential strategies are under the control of the experimenter, the experiment can be designed to explicitly study phenomena of interest.

For example, a subject participating as a middleman will keep the profit from the activities in which he or she is engaged. Subject suppliers could buy (abstract) units from the experimenter according to terms fixed by the experimenter and paying (real) money for the units. Other subjects (call them demanders) who buy from the suppliers do so because they can sell the units to the units to the experimenter according to fixed terms. The demanders make money by receiving more from the experimenter than they paid to the suppliers. The suppliers make money by paying less to the experimenter than they receive from the demanders. Of course, how the potential profits are divided between the suppliers and the demanders depend on the prices that evolve from their interactions and it is that price formation process that is studied. The experimenter knows the fixed terms given to both the suppliers and the demanders and these are the foundations for the models that will be discussed later.

As is the case of all laboratory methods, the research strategy was to focus first on the simple and special cases. From these the principles and models can be identified and improved. Once the principles or models have been isolated with some precision, the

next question was whether or not the findings are robust and whether or not they survive at a larger scale. That approach is reflected in the sequential nature of the research outlined above. The first experiments were simple and inquired first about the ability of simple markets to equilibrate and then asked about the dynamics that might be involved with the equilibration observed. Then the question turned to scalability.

Because new methodologies were an aspect of the research the research strategy focused on a relatively large number of topics. By addressing several topics rather than concentrating on just one the question of feasibility was addressed. The research answered objectively the question of whether or not experiments like those reported can be conducted. None like them had ever been attempted before. So the mere execution of any one of the experiments is at one level, a research success. The products of the research are the operational methodology, the procedures, the methodology for collecting and displaying the data and the fundamental scientific propositions identified. The broad based approach has the additional advantage of establishing the robustness of the principles used in the models. All of the research reported here rests on a family of principles found in economics and game theory. Through the study of a variety of very different environments, connected only by threads of theoretical principles, the robustness of those principles becomes established.

# 4. Technical Results

Software was successfully adapted to conduct complex experiments. In many cases research assistants could conduct the experiments without the assistance of high level programmers.

Software interface development was needed to allow experiments to be conducted by researchers without the presence of technical personnel. Market software is complex, with many modules that must be coordinated, parameterized and monitored. The task was divided into three major steps. (a) Web-based setup programs were designed and completed. (b) The user interfaces were tested against the possibilities of human error and corrected in light of experience. (c) Actual operations using the new technologies were slow to be implemented because of unexpectedly slow database operations. Because the systems interact with the database with each transaction, special programming efforts were needed to increase the speed needed for rapidly interacting markets.

Some types of multiple market experiments were successfully conducted without the help of technical personnel. Others required the presence of high level programmers during the execution of the experiment. In addition, the techniques used for development of experiment-supporting software were applied to new auction technologies and to information aggregation technologies. In both cases, research assistants have successfully implemented and conducted experiments without the help of technical personnel.

Completely separate systems of software were developed for use in the information aggregation experiments. While both markets and the parimutuel systems are continuous

time processes, the speed of the betting is much faster than are the market orders.  As a result, completely different systems were developed and implemented.

# 5. Important Findings And Conclusions

## 5.1 Vertical Linkages Among Markets

### 5.1.1 Markets with Multiple Inputs

The first experiments established the foundation for all experiments that followed. One might think that the result has been known for a hundred years but the fact is that it had not been established experimentally. The essence of the result is that equilibria in interdependent system can be observed.

> **FINDING:** *Markets with the interdependence of production, in which input prices, output prices, output and output demand are all determined endogenously, adjust toward the competitive equilibrium of the simultaneous equation, competitive equilibrium model.*

Data from the experiments conducted to establish this finding will not be included in the report because the phenomena are illustrated later in the report in the much more complex, scaled up experiments that became enabled by virtue of this finding.  Here only the theory will be introduced so the reader may understand the issues and why the question was posed and answered before more complex  economies were studied.

The essence of a complex economy is the interaction of individuals. It is the interaction and the resolution of gains from exchange and conflict as opposed to the individual acting in isolation that is fundamental from the point of view of the applications.  The general competitive equilibrium model has that interdependence at its heart.

Let $z^i = (z^i_1,.., z^i_n)$ be a vector of commodities consumed by individual i.  Let $U^i(z^i)$ be a utility representation of the preferences of individual i.  Let $z^i$ be the initial endowments held by individual i.  The classical competitive equilibrium model for an exchange economy is based on the following principles:
i.       In each of the n markets only one price exists $p = (p_1,…,p_n)$.
ii.      For each individual i an excess demand function $E^i(p, z^i) = D^i(p, z^i) - z^i$ can be derived where $z = D^i(p, z^i)$ is the solution to the problem
        max $U^i(z^i)$ subject to $p(z^i - z^i) = 0$.
iii.     Equilibrium is a price vector p such that $E(p) = \Sigma_i E^i(p, z^i) = 0$

The principle (iii) is deceptively simple in statement. It is nothing more than the law of supply and demand that is applied to the one market case in almost every textbook to illustrate the nature of price discovery.  In fact, it embodies on the order of  $nm + n - 1$ equations where m is the number of individuals and n is the number of commodities.

In spite of its seeming abstractness, the model is filled with empirical content. The prices that solve (iii) and the allocations defined by (ii) are the predictions of the model.  The

sets of equations (ii) and (iii) are the equations that the economy is supposed to solve according to theory. If the experimenter knows the equations and if the prices and allocations are observable, then the theory itself is subject to test. The question posed by experiments is whether or not prices emerge from markets that are solutions to the many equations implicit in (ii) and (iii).

The phenomenon of production can be added to the model by extending the notation and adding an agent with special preferences. Suppose some agent j is able to use some of the commodities to produce other commodities. Where $x^j = (x^j_1,\ldots,x^j_n)$ is the quantities of each of the outputs by agent j and $y^j = (y^j_1,\ldots y^j_n)$ is the vector of inputs, the agent has production possibilities governed by a function $x^j = F^j(y^j)$. The agents profits are then $p(x^j - y^j)$, which the agents wants to maximize subject to $x^j = F^j(y^j)$. The solution to this problem yields equations similar to (ii) above and quantities that must be factored into (iii) above. Thus, the addition of production to the model requires no new principles other than the specialized "utility function" of profit maximization and the associated technologies.

While the addition of production poses no problems in the technical sense of defining an equilibrium of the model, production does pose interesting problems from a system sense. As has been stated above, the model does not explain how equilibrium might be attained and in the case of production, because of the nature of the interdependencies, this is a particularly challenging issue. Output prices depend on costs, which depend on input prices, which depend on the demand for inputs, which depend on output prices. While this circularity poses no problem for a simultaneous equation, equilibrium model, it does leave open if and how this can happen in a real time environment in which information is processed only locally, by humans, and the market prices emerge from the bids, asks and competition among them.

From an empirical and experimental point of view the key parameters that are controlled in the laboratory but are not controlled in the field are the utility functions and the production functions. Modern experimental methods solved those key problems of control by using financial incentives and computer technologies. Once simple (relative to those found in the field) economies were created in the laboratory, the price formation process could be studied. Experiments demonstrated that the equilibrium can be achieved. Illustrations will be in the section in which large-scale economies are studied in the context of international trade and finance.

## 5.1.2 A Long Chain of Markets

The addition of production to the competitive equilibrium model contains elements of "chains" on markets linked in a network. The experiments reported in this section focused on if and how long "chains" of markets separated in time, space and participants might behave.

The setting can be viewed as one of location in which suppliers are located at one place and consumers are at a different location. They are connected by a series of "short" transportation hauls that must be undertaken by different transporters. No transporter can

undertake a "long haul" from seller to consumer.  Markets exist at the beginning and end of each short haul. That is, the first transporters/middlemen buy from sellers and transport to the first drop-off for sale there. At the first drop-off a different set of transporters/middlemen purchase the units in an open market and transport it to the second drop-off where a new set of transporters/middlemen have the capacity to negotiate, purchase and haul to the third drop-off, etc.

This type of environment was chosen because of the highly interdependent nature and the nature of the interdependence is related to stability analysis.  Each link of the chain is crucial to the success of the whole.  The system cannot work if it fails to work at any point.
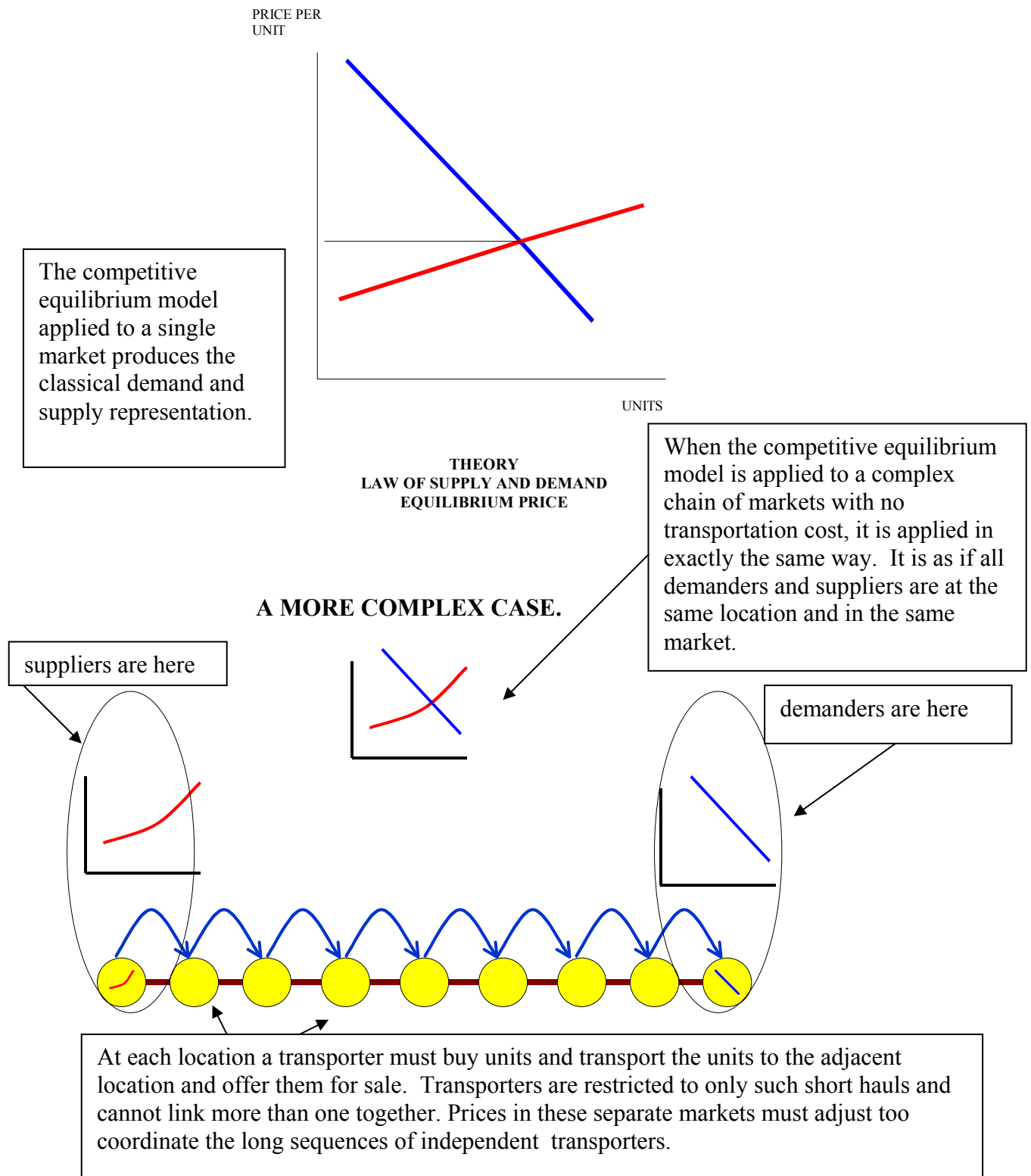
> **FINDING**: *Long chains of markets with vertical interdependencies exhibit a "backwards" process of convergence toward the competitive equilibrium.  The time flow of economic activities through such systems when the system is near equilibrium is a "lump" that moves through the chain as opposed to a continuous, equal flow.*

The nature of the observations is contained in Figure 1.  A total of three experiments were conducted.  All exhibited the same qualitative properties.

The model is summarized in Figure 2.  The classical equilibirum model of a network of markets making up a supply route holds that the prices between two markets should differ only by the transportation cost. So, if transportation costs are zero, then the prices  at the source should equal prices at the sink.  If there is a transport cost,then the price gradients will be as shown. Aggregate flow is constant across loations  according to the equilibrium model.

The dynamics of price formation in networks is illustrated in Figure 3. As can be seen, two separate dynamics are at work. Markets near the source open first, followed by the markets in the order of the chain.  Within a market and within a period the price formaiton is as shown.  The other dynamics operate across periods. The tendency is for prices across periods to equilize towards the competitive equilibrium.

While the dynamics of prices is to equalize as the system moves to an equilibrium the dynamics of flows has the opposite property as is illustrated in Figure 4.  Flows start rather constant over time but become lumpy as the system equilibrates.  All activity seems to be located at the same place and no activity anywhere else and then all activity moves to the next location. It reflects the problems and cost minimization features of coordinated activities.

9

PRICE PER
UNIT

The competitive
equilibrium model
applied to a single
market produces the
classical demand and
supply representation.

UNITS

**THEORY
LAW OF SUPPLY AND DEMAND
EQUILIBRIUM PRICE**

When the competitive equilibrium
model is applied to a complex
chain of markets with no
transportation cost, it is applied in
exactly the same way.  It is as if all
demanders and suppliers are at the
same location and in the same
market.

**A MORE COMPLEX CASE.**

suppliers are here

demanders are here

At each location a transporter must buy units and transport the units to the adjacent
location and offer them for sale.  Transporters are restricted to only such short hauls and
cannot link more than one together. Prices in these separate markets must adjust too
coordinate the long sequences of independent  transporters.

**Figure 1  Prices and Resource Flows in Chain-like Networks of Markets**

$$X = S(P_1)$$
$$P_i + T_i = P_{i+1}$$
$$D(P_9) = X$$
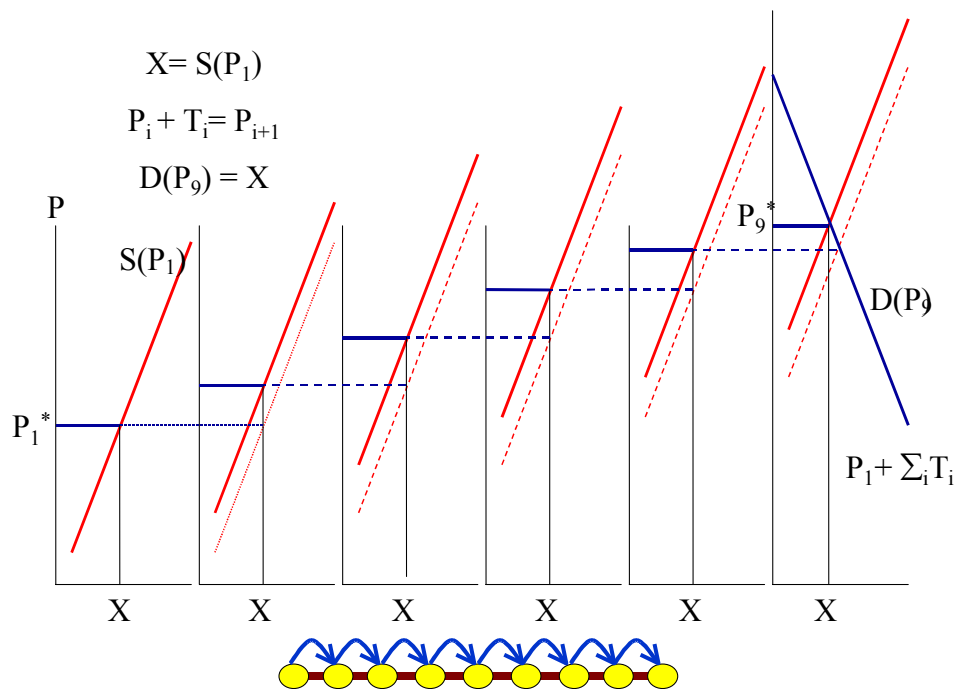
**Figure 2  The Competitive Model Applied to the Chain of Markets**



Adjustment dynamics exhibit a distinct pattern. Prices at the sink move down during a period while prices at all other locations move up during a period. The downward dynamics moves toward the source over time.

**EARLY PERIOD**

**LATE PERIOD**

prices

time

Over time the extreme prices move toward each other

**EVOLUTION OF PRICES WITHIN A PERIOD**
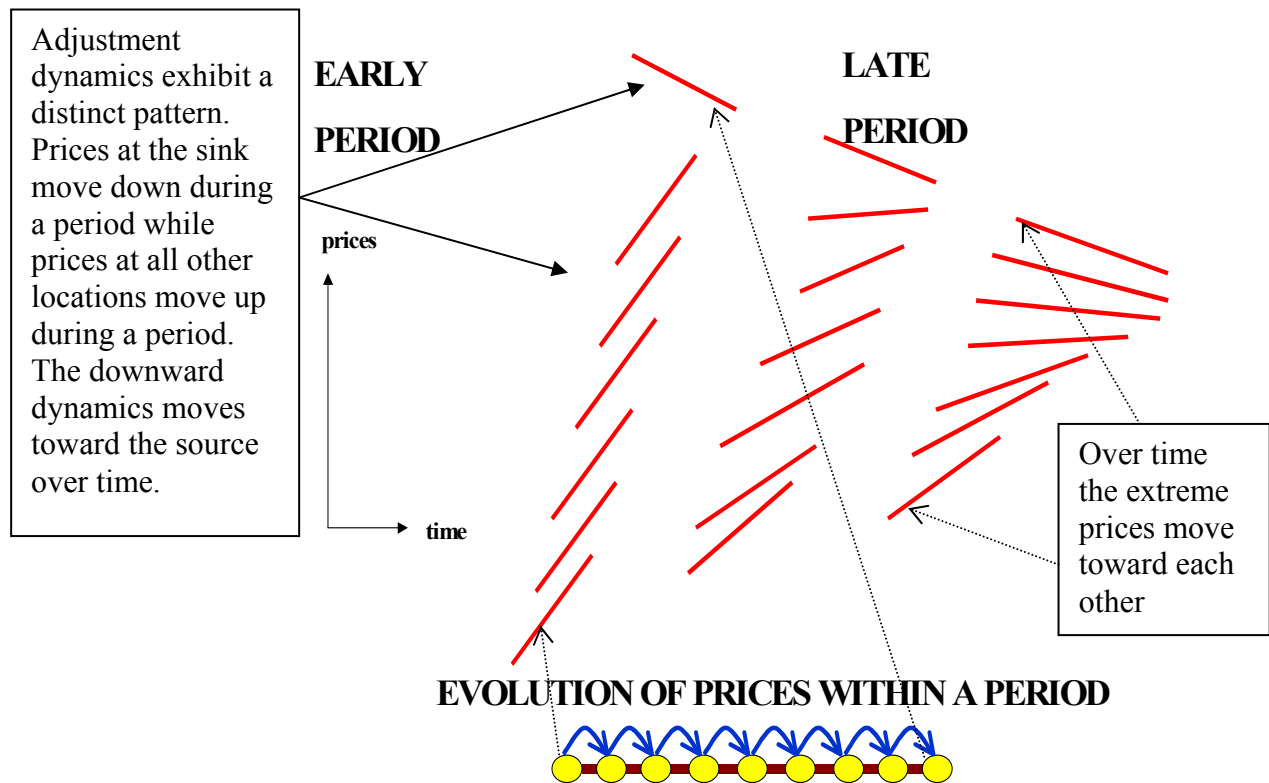
**Figure 3  Evolution of Prices within a Period**

11

EARLY
PERIOD

LATE
PERIOD

All price gradients
flatten and move
toward the equilibrium.

Price gradients

Instantaneous profile of
inventories at locations

Early, disequilibrium
flows tend to be uniform
or constant

At equilibrium the flows tend to be
lumpy.
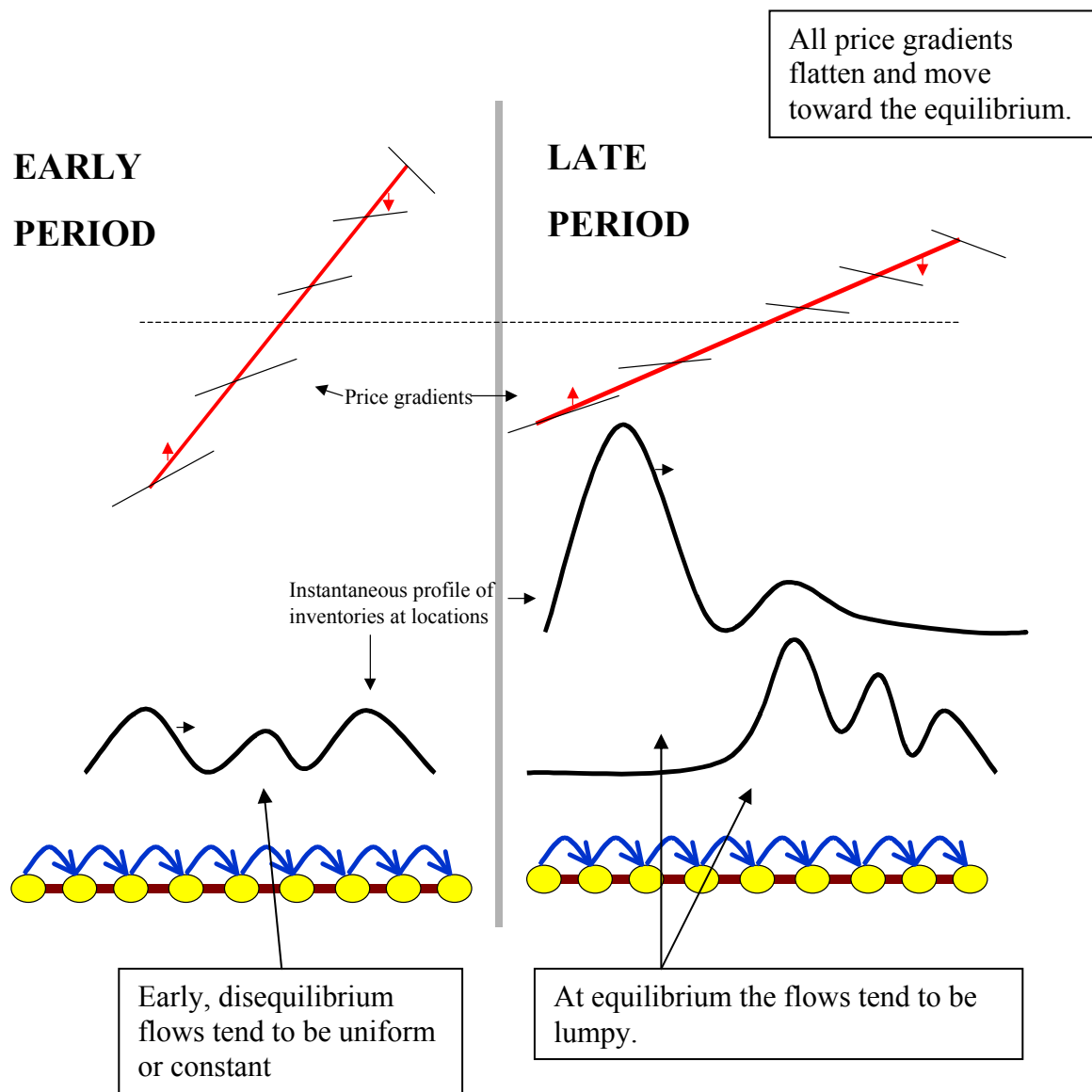
**Figure 4  The Structure of Flow Equilibration in Relation to Price Equilibration in a
Networked Chain of Markets**

## 5.2 Theories of Market Dynamics and Price Discovery

The principles of dynamics for multiple, continuous markets hold many unknowns. Previous experiments have been used to identify two different types of dynamics. One is a dynamic governing the evolution of prices and trading in a given system of markets and the other is a dynamic governing the decisions to leave one market and enter another. Whether or not both types of dynamics can be integrated into one general principle is an open question. The two dynamics seem to be substantively and operationally different things so the experiments conducted explored them both.

### 5.2.1 Markets with Complements (Depth and Open Order Book)

The classical theory of dynamics makes striking predictions that networked markets may never equilibrate. The environments that characterize networks of markets and the inherent complementarity is the source of the possibility. That is, according to the classical theory, parameters exist such that the markets never converge to the (unique) competitive equilibrium.

The scientific issue here does not stem from the probability that such parameters might be found existing naturally. The scientific issue is the reliability of any theory from which such a prediction can be deduced. Experiments in appropriate environments can thus produce excellent tests of the classical theory of dynamics. Basically, the theory says that under certain conditions prices can follow a closed cycle around the equilibrium. Furthermore, some simple and accessible parameters dictate the direction of the cycle. The surprising results are that experiments have demonstrated that many of the predictions of the classical theory can be observed, cycling behavior included.

The question posed for research here is whether or not previous experimental findings persist when the market "depth" is increased and the markets are organized with an electronic "book". These two variables are of interest because together they allow participants to acquire complements together like a package. That is, if one commodity is left shoes and the other is right shoes then if a book exists that contains standing orders to sell, an individual can purchase the pair without risk. The features remove aspects of the complementarity by, in essence, reducing the commodities from two (left shoes and right shoes) to one ( a pair of shoes). In a sense it allows immediate "production" by agents. With the complementarity removed the possibility exists that the markets would exhibit the type of dynamics different than predicted by the classical model.

The next finding is based on the results of the experiments. The finding is that the principles governing market dynamics, stability and coordination in complementary (mutually reinforcing) activities are the classical principles of dynamics even when the market institutions involve an open market book and the markets are deep. Thus, in such environments the market dynamics can prevent equilibration.

> **FINDING**: *The principles governing the dynamics of price adjustment in multiple market systems, those that operate to create instabilities, persist in the*

*presence of electronic market book and web-based market technologies. Existing experiments have been replicated and extended.*

The support for the finding is based on the classical theory of market dynamics. Continuing by using the notation developed in section 5.1.1. Let $p_j$ be the market price of commodity j. The classical model of adjustment initiated by Walras and extended by Hicks and Samuelson is summarized by the following principles:

iv.     $dp_j/dt = \Sigma_k \lambda_{jk} E_k(p)$

v.      For $k \neq j$ , $\lambda_{jk} = 0$.

vi.     For $k = j$ , $\lambda_{jk} > 0$.

An environment first studied theoretically by Herbert Scarf (1960) and extended by M. Hirota (1981) has very special (theoretical) dynamics and as such provides a good window through which to view the behavior of multiple markets. The Scarf/Hirota market environments have been studied experimentally by Anderson, Granat, Plott and Shimomura (2000) and the data reported here are replications of their discoveries.[1]

The Scarf/Hirota environment has three commodities, x, y and z with z being the numerair in terms of which all prices are quoted. Since the price of "money", the unit of exchange or "numerair", z, is defined to be one, the system must find prices for x, $P_x$, and prices for y, $P_y$. Of course both the price of x and the price of y are quoted in terms of z per unit. Three types of agents exist in equal numbers in the economy, five of each type for a total of fifteen people.

Preferences over multi-dimensional commodity spaces are induced in a direct way using monetary incentives. In the Scarf case they have a very special form. The preference maps are illustrated in Figure 5. Type I person has preference for y and z while getting no utility at all for x. Specifically Type I has the utility function (and it is the one used in the experiments) $U(y,z) = 40 \min \{y/20, z/400\}$. Type II gets utility from x an z and receives no utility from y with the utility function $U(x,z) = 40\min \{x/10,z/400\}$. Type III gets utility from only x and y, having utility function $U(x,y) = 40\min\{1/10,y/20\}$. These are the actual incentives used each period with the units denoted in cents.

The reader will notice immediately from Figure 5 that these preferences do not exhibit the gross substitute property that theoretically guarantees convergence according to the classical models of stability. However, as Hirota has shown, such a general equilibrium system will theoretically converge if the correct initial endowments exist. One such case is when each agent is endowed only with the commodity for which he/she receives no utility. Such initial endowments are points C in Figure 5.

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[1] See: Herbert A. Scarf, "Some Examples of Global Instability of the Competitive Equilibrium" *International Economic Review,*1,1960, 157-171; Masayoshi Hirota, "On the Stability of Competitive Equilibria and the Patterns of Holdings: An Example", *International Economic Review,*22, 1981,461-467; C. Anderson, S. Granat, C.R. Plott and K. Shimomira, "Global Instability in Experimental General Equilibrium: The Scarf Example" Social Science Working Paper 1086, California Institute of Technology, Pasadena, 2000.

When the principles (i), (ii) and (iii) of the general equilibrium model are applied the unique solution to the equations with strictly positive prices, is a price of x, $P_x = 40$, and a price of y, $P_y = 20$.

Figure 6 illustrates the prediction of the model under conditions in which a cycle is predicted and the figure also contains instructions for how the other figures with data can be understood. Three experiments exemplify the results. If the initial endowments are at the point CCW in Figure 5 the data should move in a counter clockwise direction. The results of an appropriate experiment are shown in Figure 7 The price patterns do exhibit the counter clockwise movement as predicted. If the initial endowments are at point CW in Figure 5 then the orbiting propensities should be in a clockwise direction and as shown in Figure 8 that is exactly what happens. If the initial endowments are at the configuration C in figure 5 then according to theory convergence will occur. As can be seen from the time series in Figure 9 the prices move almost precisely to the competitive equilibrium.

## 5.2.2 Efficiency Seeking Dynamics

The second issue of dynamics is based on decisions to enter or leave markets depending on relative profitability. In large systems the computation of the competitive equilibrium is often very complex and in an attempt to deal with the complexity modelers substitute a closely related principle of "efficiency seeking" or "surplus maximization". Once the efficient or surplus maximizing allocations are located the modeler then searches for prices that will support that allocation as an equilibrium. The implicit theory of dynamics, that markets will follow a surplus improving path is a different theory of dynamics from the one discussed in the previous section. It has not been examined well.

The experimental sessions studied and reported here were focused on the principle itself, asking if multiple interdependent markets will find the maximum surplus. To study this ten markets were created. Markets were indexed by j could be viewed as different "qualities" of a given type of commodity. The incentives were of the general functional form, $F^i(\Sigma x_j) + \Sigma \alpha_j x_j$, which was a cost function if i was a supplier and was a redemption value (i.e. "utility function" ) if i was a demander. Under these special parametric configurations, one of the markets had greater surplus than the other markets and thus it should develop, attain a competitive equilibrium and drive all other markets to close. That is, the market with the greatest surplus would drive all other markets to zero activity as producers and consumers moved to it. In summary, one can imagine several qualities of a good, all of which compete. Under the conditions of the experiment the model predicts that one of the qualities and not necessarily the "best" quality drive out all of the other qualities.

The finding of the research is as follows.

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Type
Y and Z
Preferenc

Type
X and
Preferenc
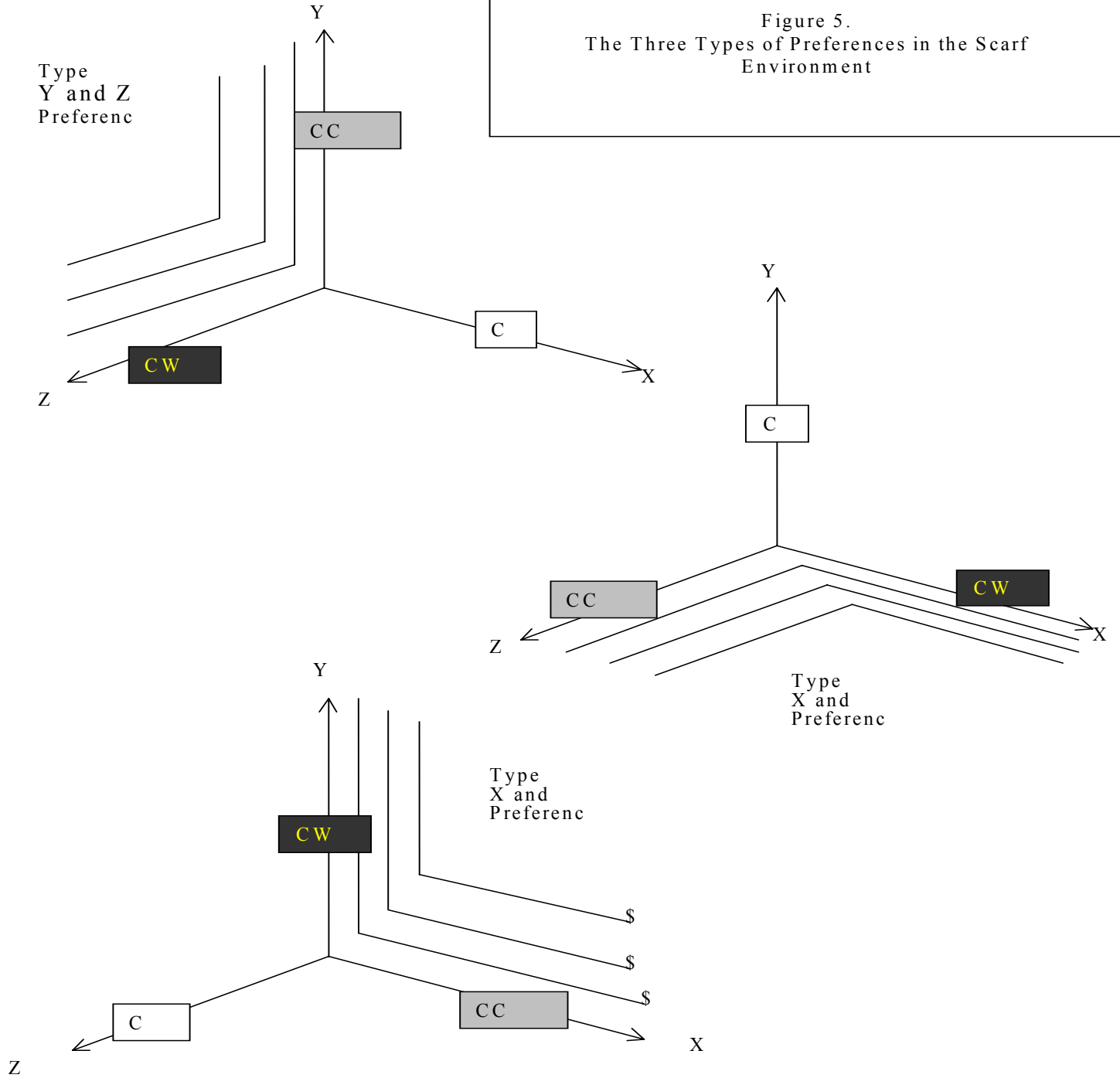
Type
X and
Preferenc

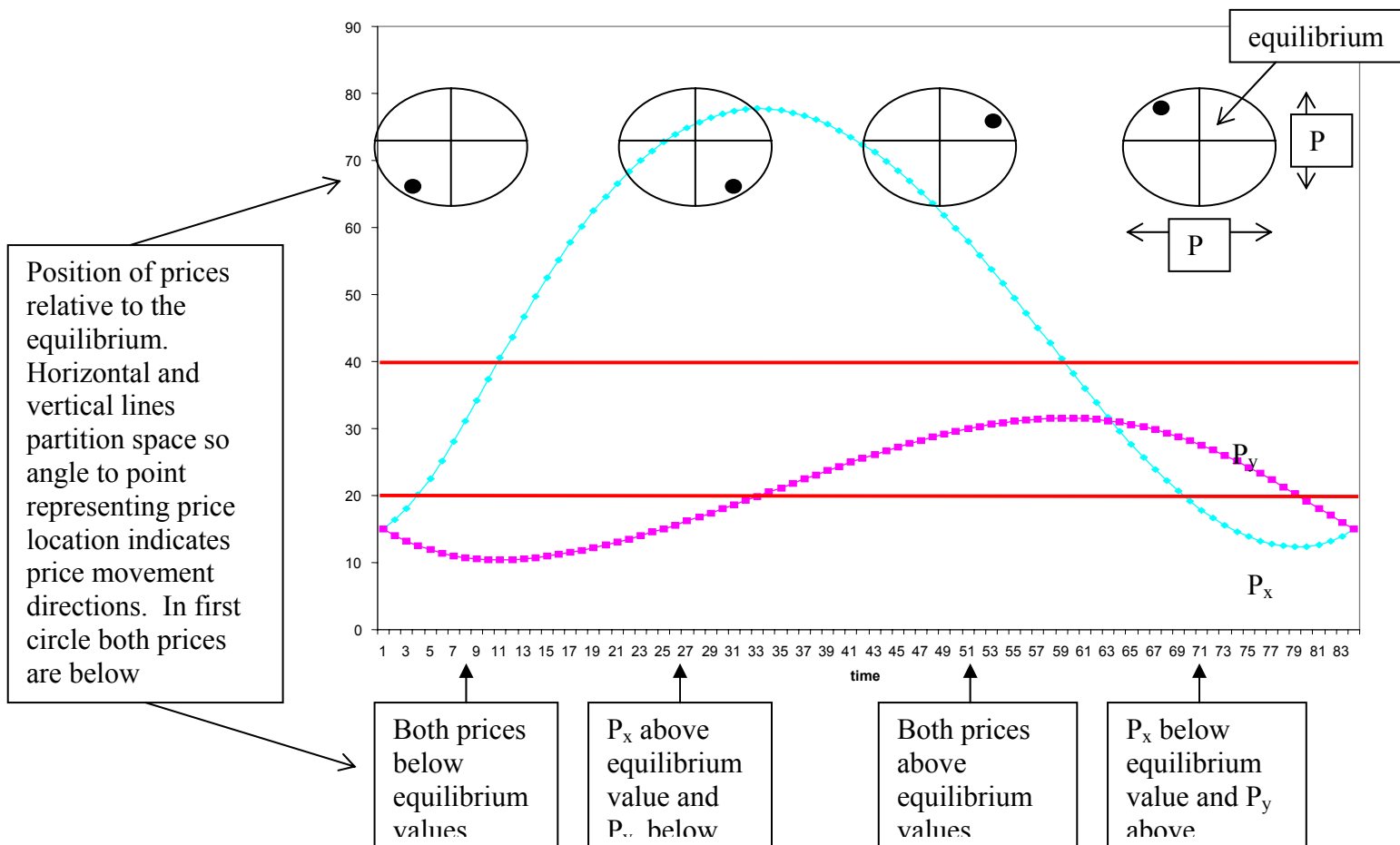**Figure 5  The Three Types of Preferences in the Scarf Environment**
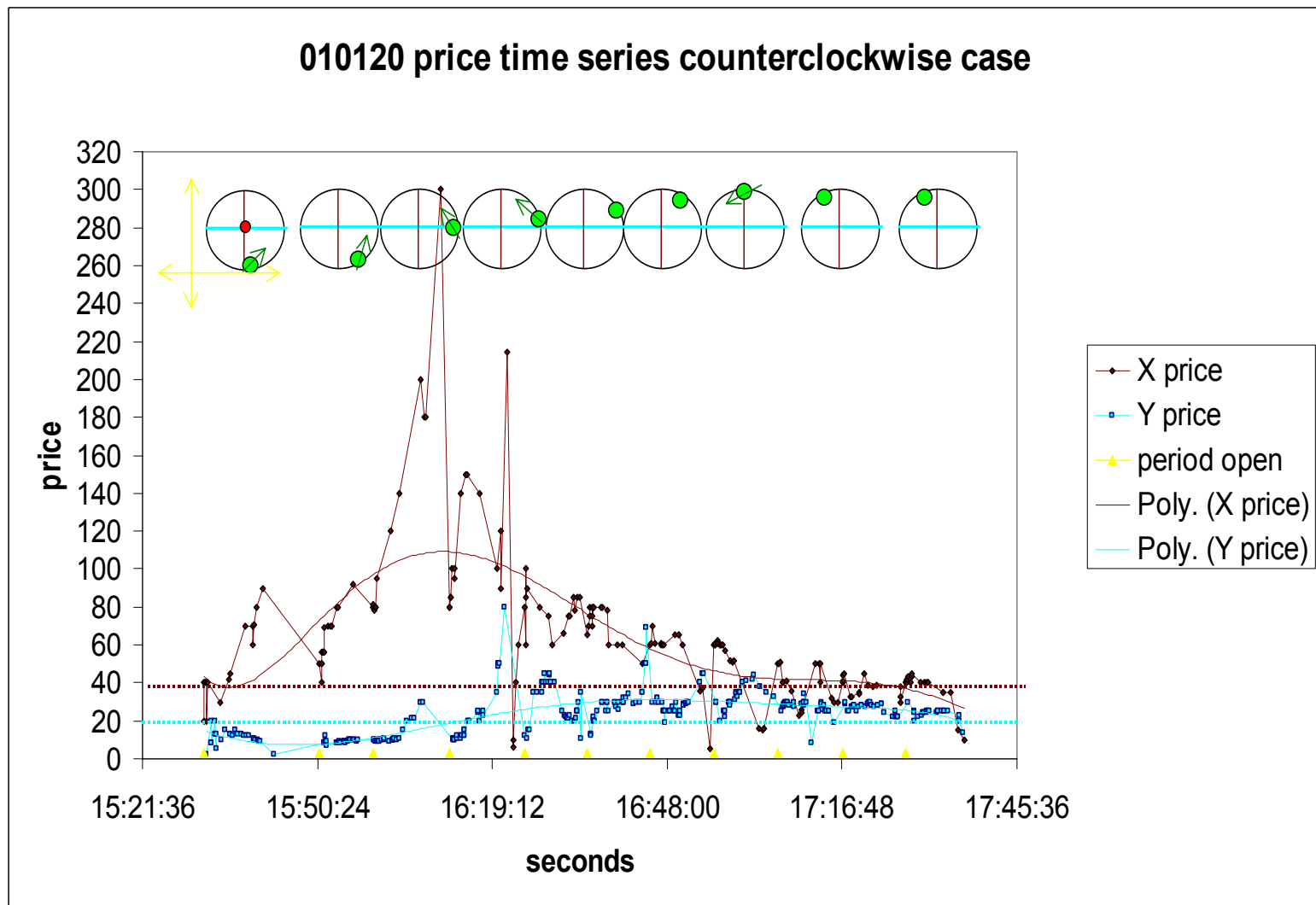
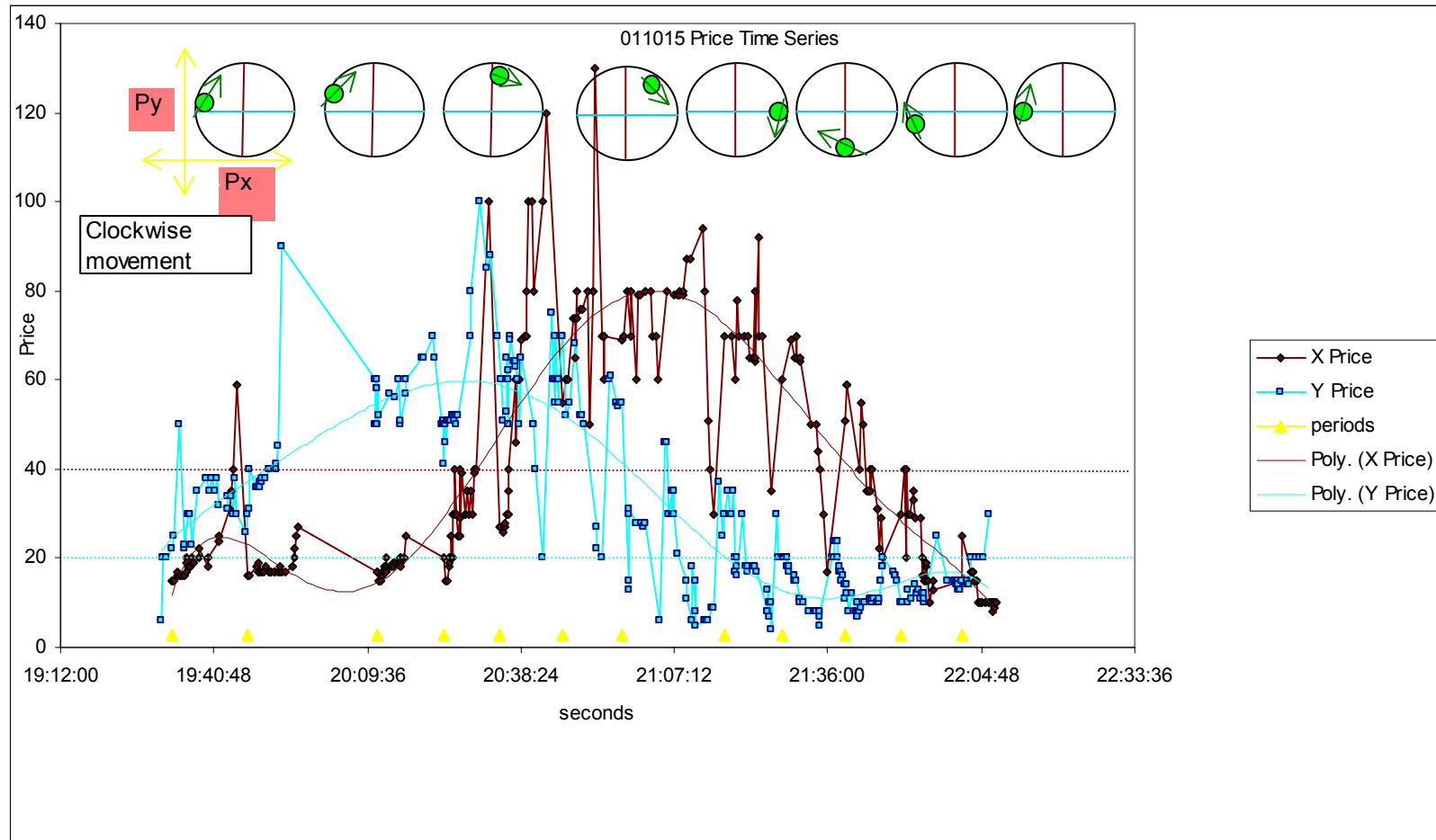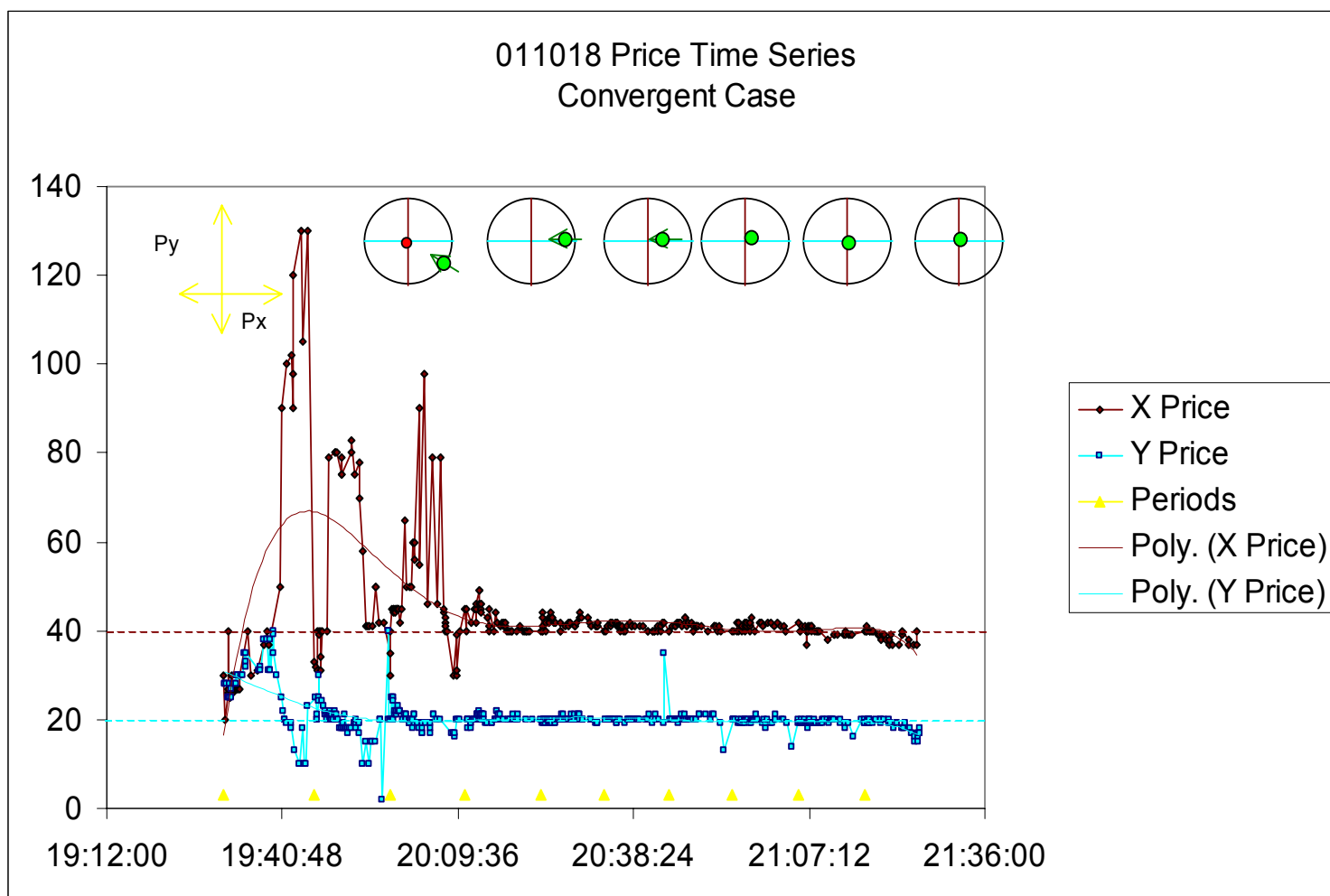**Figure 6 Orbit Model Time Series**

17

**Figure 7  Price Time Series and Polynomial Approximations of Series**

**Figure 8  Price Time Series Illustrating Clockwise Movement and Polynomial Approximations of the Series.**

19

**Figure 9  Time Series of Prices in the Scarf Environment When Initial Endowments Are at the Convergent Amounts**

**FINDING.** *A principle of surplus maximization dynamics can be observed when activities are substitutes (competitive). Economic activity flows to the market in which the surplus is greatest. Whether or not classical market dynamics can explain the results remains to be determined.*

Figure 10 shows the demand and supply for all ten markets. The one in white is the surplus maximizing commodity. The time series of trades for one experiment are shown in Figure 11, which contains the time series of all transactions that took place in all periods of all markets. As can be seen at the far left all markets were active. The "lumps" of data occur when a period is open and the sequences of lumps represent a sequence of periods. At first all markets are active. As time goes on, the market activity in all markets but market D decreases. Market D, which is the market with the greatest surplus, increases. These experiments provide support for the theory that markets move in the direction of greatest surplus.

## 5.3 Scalability of Results: International Finance and a Complex Networks of Markets

No theory of scaling exists in economics. Yet, in the issues posed here, interdependence and the ability of markets to coordinate decentralized, self-interested behavior, the effects of scale may be of fundamental importance. The research outlined above demonstrates that as a matter of principle, markets can do a remarkable job and that the reasons for success can largely be understood in terms of classical theory. Once those results have been established the natural question to pose is whether or not the results will survive as the number and nature of interdependencies are scaled up. International finance presents an excellent environment in which to initiate a test of the issues.

**FINDING.** *Experiments with economies consisting of the complexities of international finance tend to converge to the competitive equilibrium. The economies included several types of interdependencies and the economies were very large by experimental standards (21 prices, approximately 60 agents, and hundreds of equations models.*

The structure of market interdependencies and the relationship among variables are illustrated in the Figure 12. There are three countries, three different types of outputs, each of which can be produced in any or all of the countries, and there are two different types of inputs that can be used in different combinations to produce any of the three outputs. Specifically the parameters are as follow and are diagramed in the figure:

- There are three outputs X, Y, and Z, all of which can be produced in all three countries.
- Multiple inputs are required to produce each output. Production is Cobb-Douglas $f(v,w) = Av^{.25} w^{.25}$, with A equaling either 2 or 4.

**Figure 10 captures the parameters of ten different markets.** The envelope of squares traces the ten equilibrium points in the price quantity space. The white demand and supply curves, the one with the greatest quantity is the commodity that produces the highest total surplus. All markets are open simultaneously and trading takes place in all of them at first.
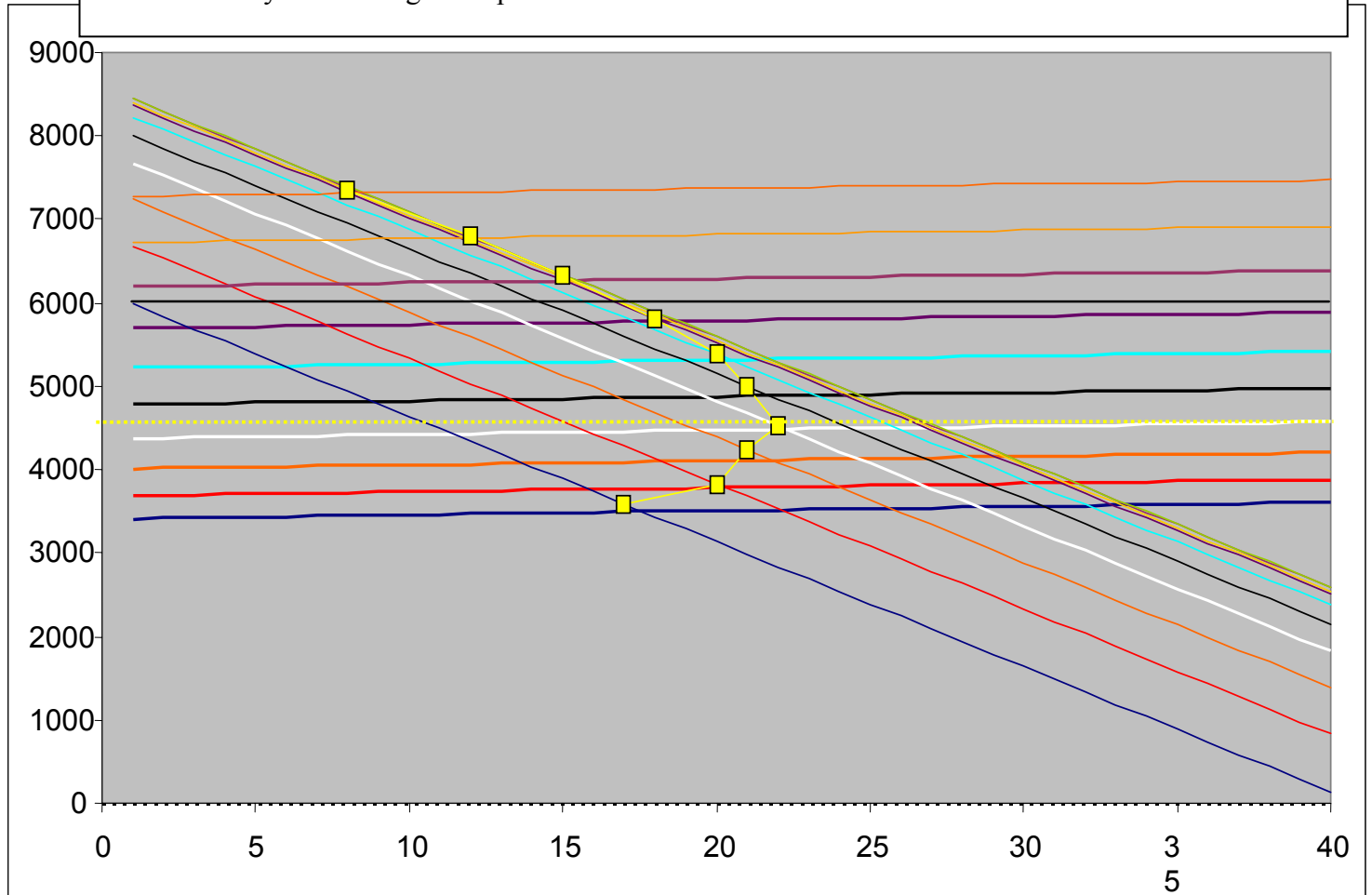
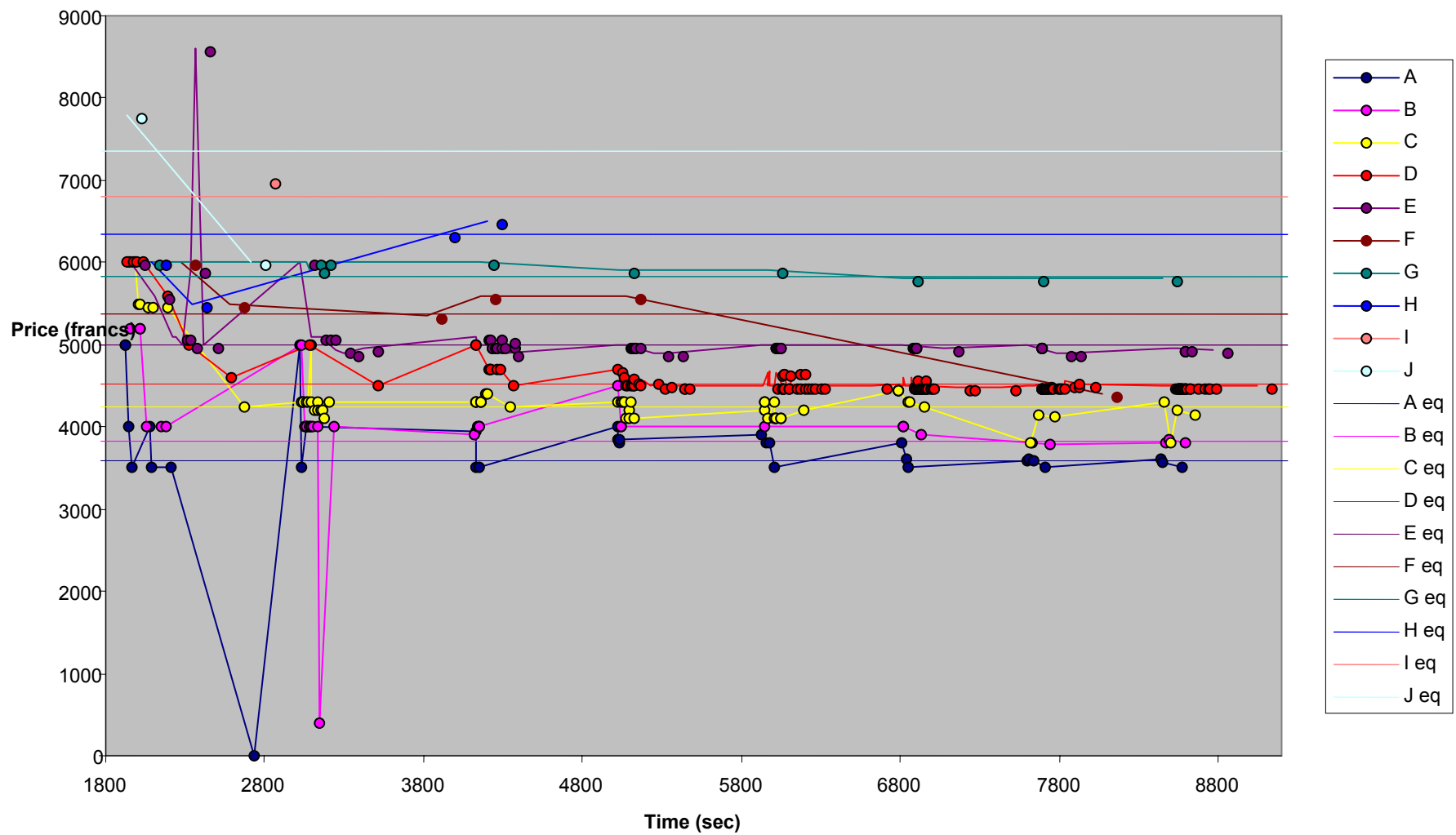**Figure 10  Parameters of Ten Different Markets**

**Figure 11  Multiple Markets, Trade Prices vs Time 020524**

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- 60 agents/subjects divided into three countries of population 20.
- Trade in each country takes place in terms of its own currency. This implies a cash in advance constraint for imports.
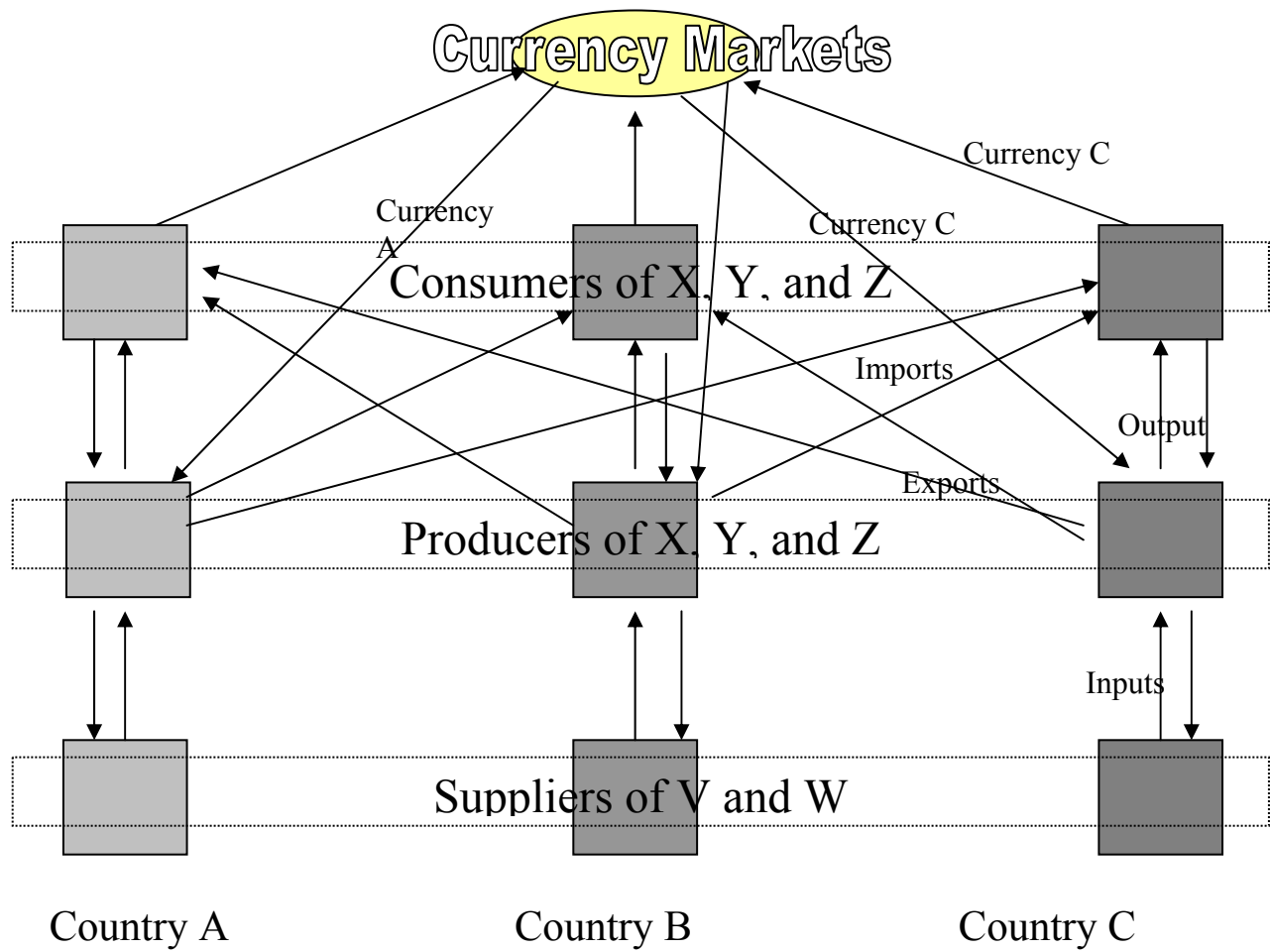- There are two inputs, V and W residing in each country. Cost of inputs is separable and linear.

The inherent complexity of the decentralized system is captured in part by the equilibrium equations summarized in the panels below. Suppose i indexes country, j output good and k input good, $IMP^{ki}_j$ = Net Imports from country k by country i of good j. $P^i_j$ is the price of good j in country i. $r^A_i$ is the exchange rate of currency i in terms of currency A (units of A's for an i). The Competitive equilibrium model involves a very large set of equations. In fact the number of equations, once each individual is considered, is on the order of 300 equations. The summarizing equations are easier to understand and they are as outlined in the paragraphs that follow.

The competitive equilibrium is the solution to the following 23 equations. The first 9 equations are the market clearing equations for each of the three outputs of each of the three countries. They say that the amount of the commodity that is consumed in the country is the net of exports of the commodity over inputs plus the home production of the commodity.

$$D^A_x(p^A_x) = S^A_x(p^A_x) - IMP^{AB}_x - IMPA^{AC}_x$$
$$D^A_y(p^A_y) = S^A_y(p^A_y) - IMP^{AB}_y - IMP^{AC}_y$$
$$D^A_z(p^A_z) = S^A_z(p^A_z) - IMP^{AB}_z - IMP^{AC}_z$$

$$D^B_x(p^B_x) = S^B_x(p^B_x) - IMP^{BA}_x - IMP^{BC}_x$$
$$D^B_y(p^B_y) = S^B_y(p^B_y) - IMP^{BA}_y - IMP^{BC}_y$$
$$D^B_z(p^B_z) = S^B_z(p^B_z) - IMP^{CB}_z - IMP^{CB}_z$$
$$D^C_x(p^C_x) = S^C_x(p^C_x) - IMP^{AC}_x - IMP^{BC}_x$$
$$D^C_y(p^C_y) = S^C_y(p^C_y) - IMP^{CB}_y - IMP^{CA}_y$$
$$D^C_z(p^C_z) = S^C_z(p^C_z) - IMP^{CA}_z - IMP^{CB}_z$$

The next 6 equations are market clearing conditions for the two inputs in the three countries. They say that the amount of the input used in the country is equal to the amount of the input that is supplied in that country. There are no exports or imports of resources in this economy.

$$D^A_l(w^A_l) = S^A_l(w^A_l)$$
$$D^A_k(w^A_k) = S^A_k(w^A_k)$$
$$D^B_l(w^B_l) = S^B_l(w^B_l)$$
$$D^B_k(w^B_k) = S^B_k(w^B_k)$$
$$D^C_l(w^C_l) = S^C_l(w^C_l)$$
$$D^C_k(w^C_k) = S^C_k(w^C_k)$$

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**Currency Markets**

Currency C

Currency A

Currency C

Consumers of X, Y, and Z

Imports

Output

Producers of X, Y, and Z

Exports

Inputs

Suppliers of V and W

Country A          Country B          Country C

**Figure 12  A Network of Markets**

The next 6 equations are the law of one price conditions. They say that prices of outputs in countries are equated by the exchange rate. There are no transportation costs in this economy.

$$p^B_x = r^B_A * p^A_x$$
$$p^B_y = r^B_A * p^A_y$$
$$p^B_z = r^B_A * p^A_z$$
$$p^C_x = r^C_A * p^A_x$$
$$p^C_y = r^C_A * p^A_y$$
$$p^C_z = r^C_A * p^A_z$$

The final three equations reflect the flow of funds theory of exchange rates. They say that exchange rates adjust so the demand for foreign exchange as dictated by imports just equals the supply of foreign exchange as dictated by exports.

$$p^A_x * (IMP^{AB}_x + IMP^{AC}_x) = p^B_y * IMP^{BA}_y / r^B_A + p^C_z * IMP^{CA}_z / r^C_A$$
$$p^B_y * (IMP^{BA}_y + IMP^{BC}_y) = p^A_x * IMP^{AB}_x * r^B_A + p^C_z * IMP^{CB}_z * r^B_C$$
$$p^C_z * (IMP^{CA}_z + IMP^{CB}_z) = p^A_x * IMP^{AC}_x * r^C_A + p^B_y * IMP^{BC}_y * r^C_B$$

The solution to the equations can be derived using common computational techniques. The question posed for experiments populated by humans interacting for cash profits through many double auction markets is whether or not the prices that emerge from the markets are the same as the prices predicted by the model – the solution to the equilibrium equations.

Several figures provide illustrations of how the complex system of price determination evolves. Figure 13 contain data on two of the three exchange rates for one of the experiments. The Figure 14 shows the output prices and input prices of one of the countries in the same experiment. The time series are the average prices in a period (like a trading day). The fixed horizontal lines are the solutions to the equations for that particular variable.

The overall properties exhibited are as follows:
- Convergence toward the general competitive equilibrium model is observed.
- The path of convergence seems to be from the "autarky" or "no trade equilibrium" to the efficient equilibrium.
- Uncertainty and thin exchange markets seems to inhibit international trade.
- Input prices development lags the output prices.

Of course the major news is that all parts of the economy functioned roughly as theory suggests. Thus the properties of behavior observed in the smaller experiments were substantially preserved at a much larger scale.
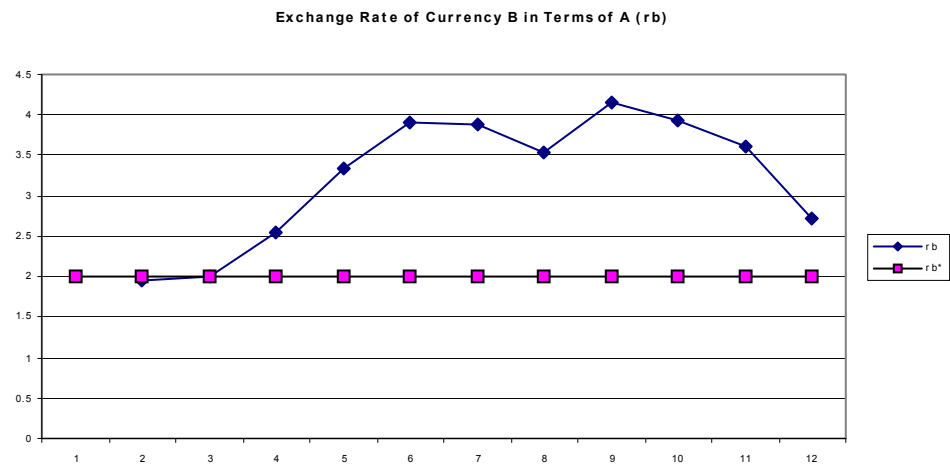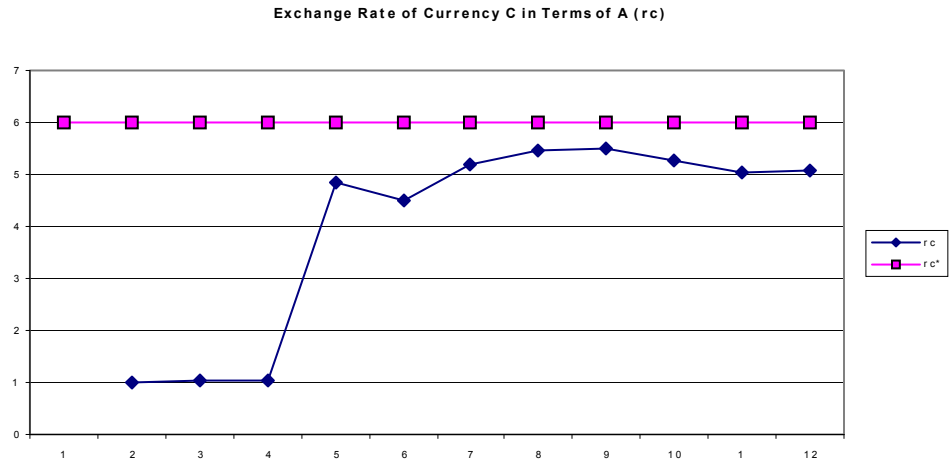
Perhaps the most important conclusion is that the technology and procedures needed to conduct this type of complex system experiment with real people exist and work. The principle of equilibration observed in simpler economies can be observed in the more complex international economies. The patterns of data do suggest that equilibrium models will show inaccuracies when applied both to laboratory and to field data, because the equilibration process may be lengthy. Systems of interdependent markets do not equilibrate immediately. Equilibration seems particularly slow in factor markets with derived demand.
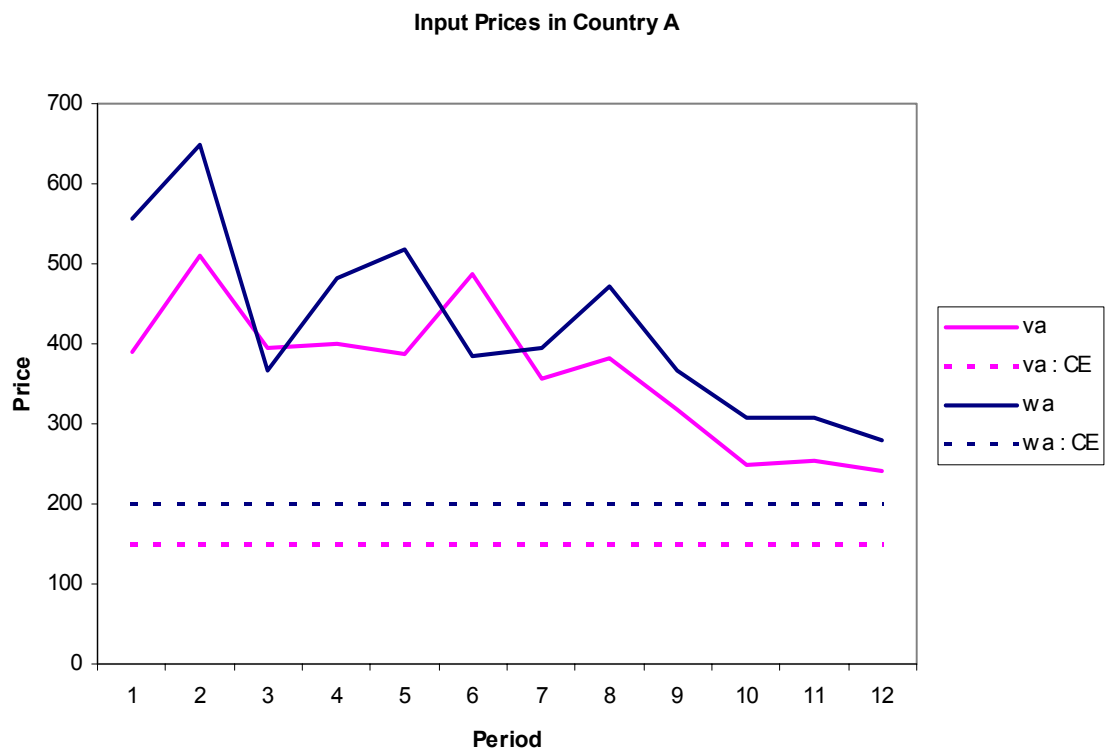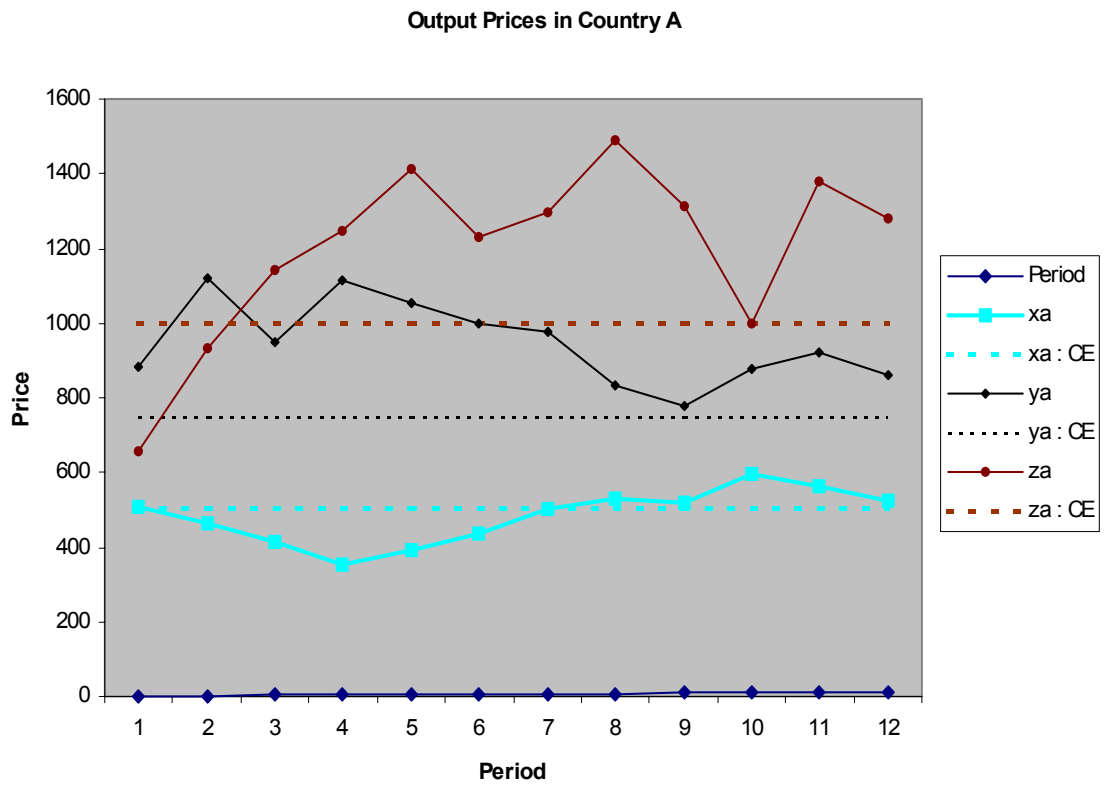
## 5.4 Information Aggregation

Basic research on markets and market processes has produced evidence that markets can aggregate information. That is, "soft" information that is dispersed in the system as subjective hints, opinions and intuitions can sometimes be transformed into hard data by the application of proper communication and incentive systems. The intuition follows from the every day experiences in which we infer information from the behavior of others. For example, when finding a place to eat dinner one often looks for the number of customers. If you stop and look in a window on a busy sidewalk others will pause and look as well. Fishermen will look for other fishermen. After watching the increase in the price of a certain stock one learns that there was an earnings increase or other good news.

In each case the behavior of others carries useful information about what they know. The behavior of systems tells us something about the information of those who are using the system. A system that is designed specifically for the purpose of collecting information is called an Information Aggregation Mechanism. The key idea is to provide individuals with an action that captures beliefs and give them incentives to take such actions. If an observing individual knows something about the actions and the interests of those taking the actions then the observing individual can deduce something about information on which the actions were based. The question posed for research is whether or not new forms of systems, which have as their sole purpose the gathering of information, can be successfully designed and implemented. The research starts with the observation that the odds that emerge in parimutuel betting systems at horse races are reasonably good predictors or the winner of the race. The question posed is whether a modified version can be designed to become an Information Aggregation Mechanism.

The creation and testing of Information Aggregation Mechanisms involves standard methodologies of experimental economics. Partial information, signals so to speak, were distributed to individuals in a manner that individuals may have a subjective probability over various states of the world but no one knows for certain. If the signals received by all were pooled the state could be estimated with only a small probability of error. Individuals take actions based on their private information and the actions they see others taking. In the case of a parimutuel like system the action would be placing a bet for real money, enough to make it interesting to the individual. The emerging odds can be used as predictors of the state.

**Exchange Rate of Currency C in Terms of A ( r c )**



**Exchange Rate of Currency B in Terms of A ( r b )**



**Figure 13  Time Series of the Two Exchange Rates as Seen From the Point of View of Country A**

## Output Prices in Country A



## Input Prices in Country A



**Figure 14  Time Series of Two Output Prices in Country A and  Two Input Prices in Country A**

**FINDING:** Specific modifications of a parimutuel system of betting can transform it into an effective Information Aggregation Mechanism, capable of collecting and aggregating information similar to markets and capable of indicating when the information it contains is poor.

Early work on information aggregation mechanisms demonstrated that parimutuel-like systems demonstrated the feasibility and initiated both estimates of accuracy and performance measures. While the success rate is high given the difficulty of the task, the efficiency of performance is reduced by natural strategic considerations. Risk averse people will not place bets while other people bluff and wait until the last minute to place bets thereby reducing, confusing and delaying the information. In addition, the early odds reflect disequilibrium bets that produce a bias in the overall probability measures. In spite of the problems, the system produces remarkably good predictions but the room for improvement is obvious.

The question posed for research was whether or not an improved system could be created. Two different designs were developed and tested.

Design 1 made two fundamental changes in the first parimutuel IAM. First, the price of tickets was made an increasing function of time elapsed from the beginning of the betting period. Thus bets placed early were cheaper than bets placed later thereby providing an incentive to get information into the system early. Individuals adjust and update their beliefs in the light of the actions taken by others. While this type of updating has the capacity to greatly improve the operations of the IAM. Of course, it also has the capacity to create "mirages", bubble like speculations that lead people to believe the system has achieved an information aggregating equilibrium when in fact it has found the "wrong" equilibrium. The second change was to repeat the betting process after an individual had seen the bets placed but before they saw the state of nature. So, the new bets were placed after they saw the old bets, and could update their beliefs. Thus the system produced two predictions from the two different rounds of betting. According to theory, the odds that emerge form the second round would not contain the disequilibrium bets that existed in the first round. Thus in the second round of betting the biases would be removed.

Experiments with Design 1 demonstrated that the accuracy of the system was definitely improved. However, two additional facts were learned. The mirages and inaccuracies of the system were frequently coincident with the existence of poor information that needed to be aggregated. Basically, when the information dispersed in the population is poor the system would wander off into a mirage (bubble) and predict with great confidence (attach a probability near one to the state) a state that was the wrong state.

The second lesson was that when the information dispersed in the population was poor, the individuals who were betting seemed to know that the information was poor. Something about the patterns of betting gave them a clue. The risk averse people reacting to those hints would place no bets, thereby blocking the use of whatever information they had, reducing information quality even more. We do not know the

exact vehicle that alerted the subjects but we discovered that the information is contained in the variability of the bids that take place early. Erratic movement over the states of early odds relative to such movements in other periods signals the likelihood of poor information. That is, erratic odds movements that occur early indicate the lack of a strong signal to be captured by the system. These phenomena can be identified in the entropy of the early odds and used as a measure of the state of the system. When the entropy of the system is high the system is indicating that the information to be aggregated is not especially reliable.

Design 2 involved one fundamental change designed to test a "warning" that the underlying information is weak. This was accomplished by adding a "first stage, sealed bet" parimutuel system. Agents were given a fixed number of valueless tokens to bet in a fist stage parimutuel. The "house" created a prize to be paid to the holders of tickets on the winning state. The bets were placed privately and not revealed until all bets were placed. Since the tickets were worthless, aside from their use in betting, even the most risk-averse person would bet all since there was nothing to lose and something to gain. Once all bets were placed the first stage odds were revealed and the betting was open for the actual parimutuel.

The entropy of the first stage odds carries information about the quality of the underlying signals. High variability means low agreement on signals, which suggests poor information to be aggregated. In the two-stage system the overall predictive ability of the IAM increased and some of the mirage/bubbles were removed. When mirage/bubbles are developing the system itself signals that the prediction has entered an environment in which the reliability of its predictions might be compromised.

The nature of the information aggregation problem is illustrated in Figure 15. The methodology of experimental control, achieved by distributing information that is known to the experimenter, is illustrated in Figure 16. Each individual receives a small sample conditional on the randomly chosen state. Any particular individual knows very little but the collection of data distributed to all individuals can amount to a strong signal. That is, when the signals distributed to all individuals is pooled, and Bayes law is applied, a strong signal is extracted and used by the experimenter as a measure of the accuracy of the IAM. Of course participants cannot communicate with each other at all so only the information they get is from the signal privately given to them by the experimenter and the pattern of betting.

The bets placed through the parimutuel betting system are on the underlying states and real money is at stake. The timeline of events is contained in Figure 17. As betting begins the odds are displayed in continuous time. Figure 18 contains the time series of normalized prices (odds inverted) for one of the very first experiments. As can be seen the odds develop slowly. In this particular parimutuel the cost of tickets was constant and the betting window closed at some randomly chosen time after an announced "near end". As can be seen in the figure, aggressive betting takes place only near the end of the period. Eventually a state emerges as the winner according to the "group opinion." It is the state with the lowest odds – the highest "normalized price."

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The "group opinion" can be wrong. The normalized price for the wrong state could be near 100%, indicating that the system is suggesting that the group knows the state with near certainty even though it is wrong. Such "bubbles" tend to occur when the information in the system is not good. Participants seem to sense when it is happening. Figure 19 illustrates the phenomena.

Modifications of the parimutuel system improve the accuracy dramatically. The modifications get the information into the system quickly, remove some opportunities for "bluffing" and also include a sealed bet stage that signal situations when much disagreement exists and therefore weak signals are possible. Figure 20 captures the comparisons.

# 6. Significant Developments

See the section above.

# 7. Special Comments

See the sections above.

# 8. Implications for Further Research

This project opens for research several areas that show considerable promise.

1. The robustness of classical laws of market behavior were explored in the context of a wide range of different network interdependencies. The principles seem to be correct but the application and modification of those principles to situations that are of importance has yet to be done.

2. It is clear that models of dynamics need more attention. In fact the experiments demonstrate that the actual behavioral laws reside in the dynamics as opposed to the equilibrium itself. Much more work on the dynamics, the nature of lags and anticipations is needed. Basically, a theory of disequilibrium is required.

3. Special institutions that cause impediments in systems are in need of study. Price controls and incomplete markets can be imposed in experiments. The experiments reported here provide a baseline for exploring the implications of such important institutions.

4. The details of information in markets and information transmission in markets are in need of more study. Specifically, markets appear to be finding the solution to a large set of nonlinear, simultaneous equations. However, information about those equations is severely limited. How is the appropriate information entering the market? The answer is unknown.

5. It is clearly possible to create systems that have as the sole purpose the collection and aggregation of information.  The results reported here demonstrate that it is possible to dramatically improve the functioning of one such system. Other systems, other patterns of information and interdependencies need to be studied.  The experimental testbed approach is clearly successful. It should be used with greater intensity.

6. Aspects of the data suggest that principles of dynamics might be extendable to the formation and evolution of networks. Experiments and theory along such lines would be valuable so we can better understand how to use models of the existence of networks, the forms of networks and the breakup of networks.
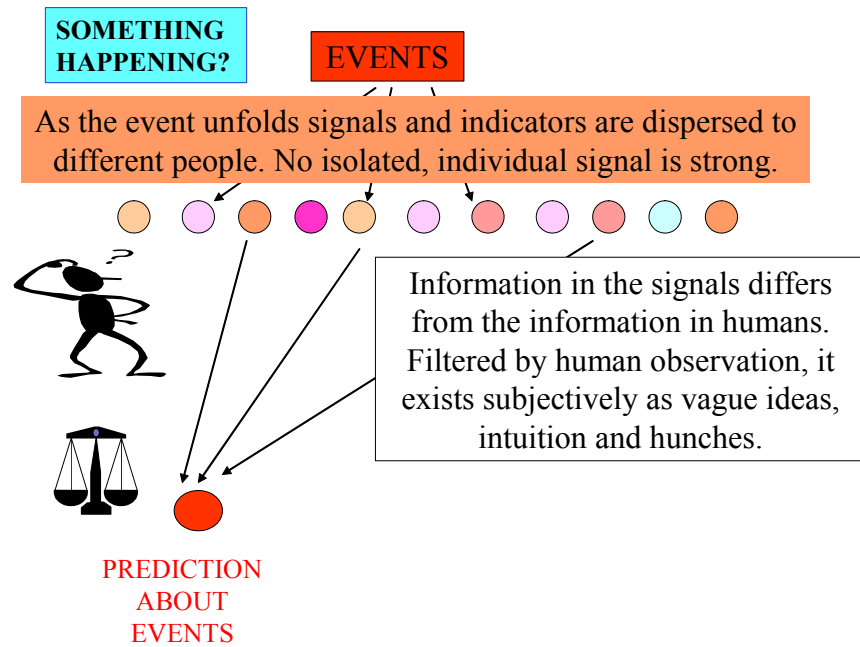
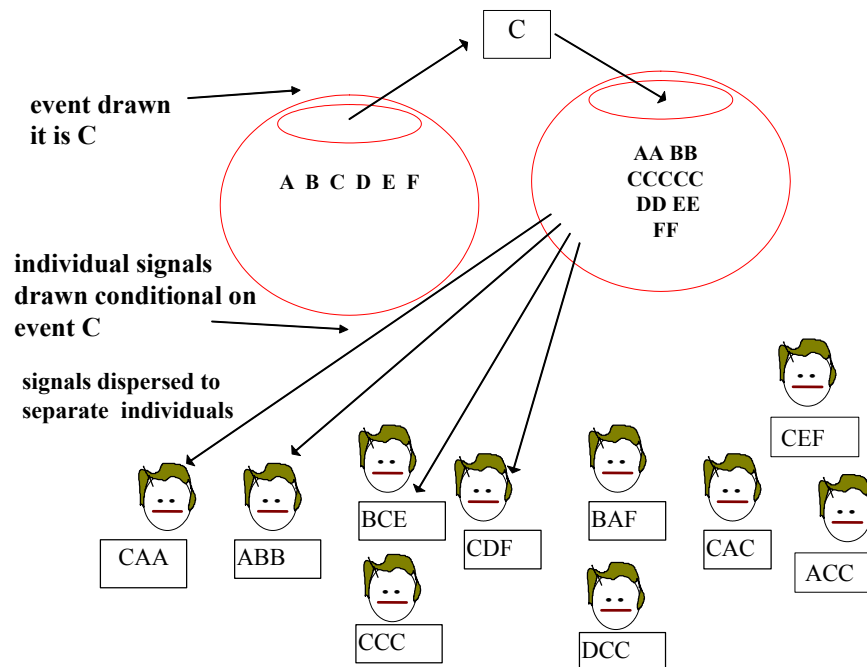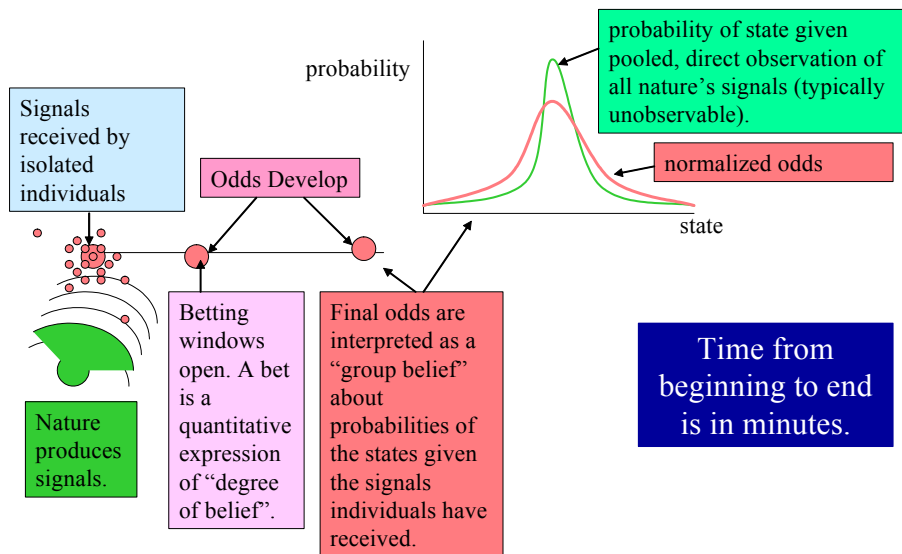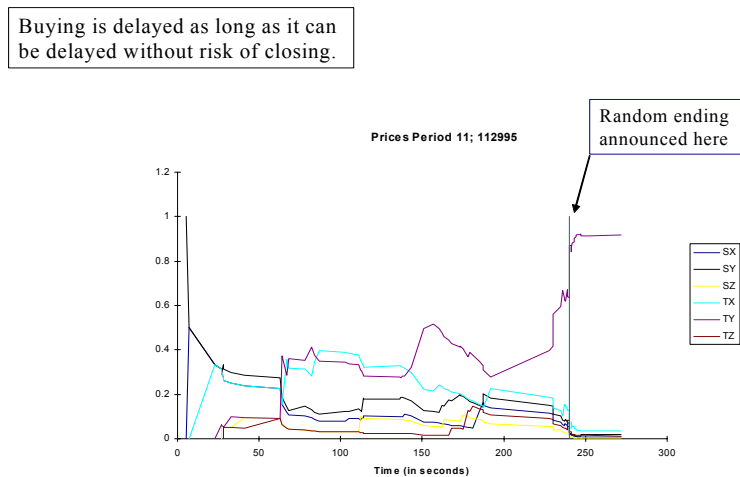**Figure 15  Information Aggregation Mechanisms**



**Figure 16  Information Privately Distributed to Individuals in an Experiment**

**Figure 17  A Parimutuel Betting System Designed as an Information Aggregation Mechanism:  A Network for Collecting and Organizing "Soft" Information**



**Figure 18  Time series of implicit prices (one over odds) for each state**

**Entropy of observed odds as an indicator of a less reliable prediction.**

Tests were divided into those for which the mechanism produced a relatively poor prediction and those for which the mechanism produced a relatively good prediction. From each set, the entropy of the observed odds was calculated as a function of time.

The less reliable predictions are those for which the most likely event (according to prediction) was not the most likely event (according to the pooled data). The more reliable predictions are those for which the modes are the same.
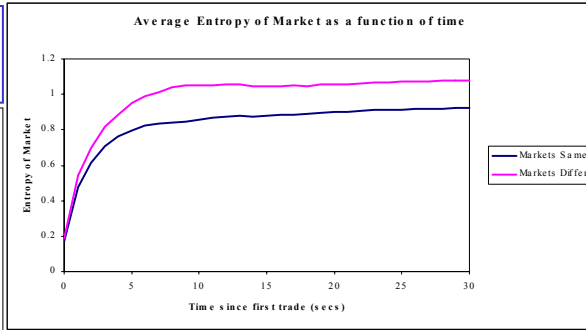


Unreliable predictions are accompanied by higher entropy of odds throughout the betting period.

If the variability of the odds, the entropy that results from betting, is high then the information that is being produced by the system is less reliable.

It should be noted, however, that even the poorest of predictions is better than predictions based on decisions based on prior information of equally likely events.

**Figure 19  Reliability Detection**
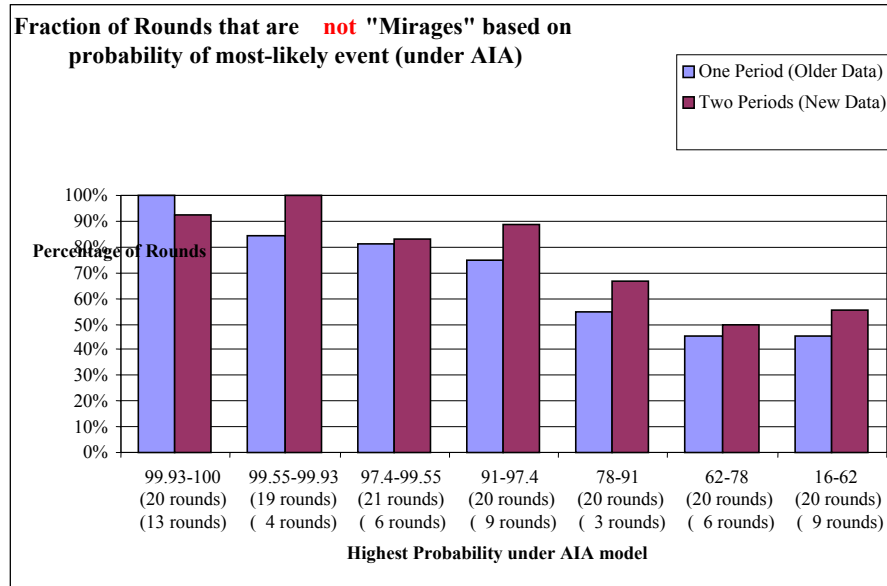


**Figure 20  Regardless of the height of the mode of existing information the two period process does a better job of avoiding mirages**